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HEAT TRANSMISSION BY RADIATION, CONDUCTION AND CONVECTION

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ON THE APPLICATION OF SCIENCE
TO THE USEFUL ARTS
1834–1845

HEAT TRANSMISSION BY RADIATION CONDUCTION AND CONVECTION

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HEAT TRANSMISSION

BY RADIATION, CONDUCTION AND CONVECTION

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BY

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PREFACE

It is hardly necessary to demonstrate to engineers the fundamental importance of the subject of heat transmission. Whilst the laws concerning the conduction of heat have been fairly well known for a long time, the laws of radiation and convection of heat are only now becoming more clearly understood. Although physicists have studied and experimented upon radiation and conduction of heat and formulated their laws, a perusal of our text-books on heat and on the properties of matter makes it apparent that physicists and chemists generally have almost completely neglected the equally important laws of convection, and the enunciation and verification of these laws of convection have been due mostly to the work of engineers. As early as 1874 Professor Osborne Reynolds showed how unsatisfactory is the common notion that the exchange of heat between a flowing fluid and a surface is dependent only upon the difference of temperature and the time of "contact." His theory was first published in a paper he read before the Literary and Philosophical Society of Manchester in 1874, and by permission this paper has been reproduced on pages 111 to 115.

As a student of engineering under Professor Reynolds the writer was naturally made conversant with this theory of convection, but experimental confirmation of a satisfactory nature was necessary before engineers could feel much confidence in the scope of its application. In experimenting and in collecting experimental data the writer was not concerned so much with proving the Reynolds theory as with testing it, and he therefore leaves the results described and discussed in Chapter IV to speak for themselves. These results also furnish data and particulars suitable for use in the design of heat transmission plant.

The laws concerning convection and the resistance to the flow of fluids have been discussed in Chapter III at some length in view of the intimate relations existing between the resistance to flow and the transmission of heat by convection. Also in describing apparatus and experimental data in Chapters II, III, and IV, a fair amount of detail has been given so as not to leave the evidence and information obscure. As far as possible all the experimental points have been plotted in the various graphs, as this was felt to be necessary so that the reader could see how near the curves follow the points, besides indicating the number of separate tests made in each series.

Nearly the whole of the experimental data, however, were obtained from small apparatus. Comparisons with the results of tests on boilers, condensers, and similar plant, and the application of the laws of heat transmission to design, have been reserved for treatment in *Heat Transmission in Boilers, Condensers, and Evaporators.*

Special acknowledgment is due to the Institution of Civil Engineers, Institution of Mechanical Engineers, and the Institution of Engineers and Shipbuilders in Scotland for permission to use illustrations and information from their several *Proceedings*, as well as to the several American and other publications mentioned in the text. The author also desires to acknowledge his indebtedness to Professor A. L. Mellanby for many facilities granted during the course of the author's experiments and investigations on heat transmission, and to Professor A. H. Gibson for permission to use in Chapter III some illustrations from his book on *Hydraulics and its Applications.*

R. ROYDS.

DUNDEE,
October, 1920.

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HEAT TRANSMISSION
BY RADIATION, CONDUCTION AND CONVECTION

CHAPTER I

RADIATION OF HEAT

Transmission of Heat.—According to our accepted ideas there are three methods by which heat may be transmitted, viz. by radiation, by conduction, and by convection. For example, the heat and light which reach us from the sun across the intervening space is emitted by the sun as radiations. Again, by conduction heat is transmitted from hot to colder parts of a body without the transference of the parts of the body. With convection, however, the heat is carried from place to place by the bodily transference of the particles of matter with which the heat is associated. In engineering practice it is commonly found that all of these methods of heat transmission are actively utilised, as, for instance, in steam boilers, where heat is radiated by the fuel and flames in the furnace, is transmitted through the boiler plates or tubes by conduction, and heat is also transmitted by convection from the gases to near the metal surface and from near the other surface of the metal to the water. With these few words of introduction each method will be discussed in turn with particular reference to the transmission of heat in boilers, condensers, and similar apparatus.

Radiation.—The phenomena of heat and light are essentially associated with matter. According to the accepted theories of the constitution of matter it is assumed that all matter is made up of a very large number of indefinitely small particles, named molecules, which are more or less independent of one another when the matter is in the gaseous state, but which in the liquid and the solid states of matter have lost some part of their independent character. The molecules are also supposed to be subdivisible into atoms. These atoms and molecules, or at any rate some portions of them, are further supposed to be in a violent state of vibration, oscillation, or agitation which is associated with what we call heat and light. It is assumed that these vibrations or oscillations set up waves which travel at an enormous velocity, something of the order of 186,000 miles per second in vacuo. These radiations, however, only become evident to our senses when the waves impinge on sympathetic particles or molecules of matter. That is, if the molecules are so constituted that they, or some portions of them, respond to the impinging waves, or, in other words, if their natural period of vibration or motion is in "tune" with these waves, the waves are extinguished and vibration is induced. Otherwise the waves pass on or are reflected, apparently leaving the molecules unaffected.

A radiating body, however, being composed of innumerable molecules, each with vibrations according to its nature and temperature, generally sends out radiations of various wave-lengths. An incandescent body, for instance, not only sends out light of various wave-lengths but also heat, which has wave-lengths relatively longer than those of light. In any case, the radiations of light and of heat are similar, those waves constituting light being short, those of heat relatively long. Conclusive evidence of this similarity is furnished by the fact that heat radiations obey the same laws of reflection, refraction, polarisation, and interference as light radiations. The apparent difference merely arises from the fact that they affect our senses differently.

From what has been said, then, it will be readily understood that a body is capable of absorbing its own radiations. The radiations emitted by a uniformly constituted hot gas, for instance, if allowed to fall on gas of the same constitution would be absorbed by the latter. Thus it is that the radiation of heat from a flaming gas comes principally from the surface of the flame, since the greater part of the radiations from the interior of the flame are probably absorbed by the surrounding gas of the same constitution, and therefore do not penetrate to the surface of the flame. It should be noted, however, that the greater the mass of the gas the longer will the body of the gas preserve the temperature or the intensity of the radiations.

The Theory of Exchanges.*—According to theory the molecules of a body would only be reduced to a state of rest if cooled down to the absolute zero of temperature, −273° C. (−460° F. nearly). Thus, at all temperatures met with in common experience the molecules, being in a state of vibration or agitation, set up radiation waves to an extent depending upon the temperature and nature of the molecules. They also absorb radiations to which it is their nature to respond. Therefore when a body has acquired a constant temperature and is not in contact with other matter the energy which is being radiated is equalled by the amount being absorbed and thus an equilibrium is maintained. In this manner, then, a system of bodies contained in an enclosure impervious to heat would ultimately attain a condition of equilibrium of temperature.

Suppose a thermometer is suspended in a warm room at a steady temperature. If the thermometer has attained the steady temperature condition, the heat it absorbs from its surroundings is balanced by an equal amount being lost. But if a colder body be now brought into the room, say, a lump of ice, and placed in the neighbourhood of the thermometer, the thermometer becomes screened from some of the radiations previously falling on it, and being now subjected to radiations from a colder body, the total radiations received or absorbed by the thermometer would diminish and the thermometer would fall in temperature until it attained a new condition of equilibrium. This explains why a thermometer, unless properly screened, cannot record the true temperature when placed in a furnace in which the walls or some portions of the walls are at a lower temperature than the furnace, as in a locomotive firebox for instance. In such a case, a thermometer bulb of whatever type radiates and absorbs heat much more readily than the flame or gases in which it is immersed, and the radiations to and from the thermometer are readily transmitted without much being absorbed by the gases. The colder walls absorb more radiant heat than they emit, and therefore, if the thermometer can " see " the cold walls, it radiates more heat than it receives by radiation from these walls, with the result that it tends to fall in temperature. This loss of heat from the thermometer has to be made up mostly by the heat it absorbs due to radiation and convection from the gases ; but this supply necessitates a difference of temperature between the gases and the thermometer. Instances of temperature errors due to this cause are given on pp. 18 to 21.

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* The idea of exchanges was introduced by Prevost of Geneva in 1792.

Returning to the idea of bodies in an enclosure which is impervious to heat and therefore at a uniform temperature, and considering that the radiations falling on any body within the enclosure will either be absorbed, reflected, or transmitted, it follows that any radiations not absorbed are either reflected or transmitted. If the body is opaque none will be transmitted, and it is evident that what is not absorbed will be reflected from the surface of the body. Therefore a good reflector will be a poor absorber. Further, when a constant temperature condition is attained, the amount absorbed is equal to that emitted ; thus a body which radiates feebly is also a poor absorber, which, if the body is opaque, also means that it is a good reflector. Conversely, an opaque body which is a poor reflector will be a good absorber and radiator. If the transference or transmission of radiations takes place readily through a body it follows that it will be a poor absorber, radiator, or reflector under the particular conditions in which the radiations are readily transmitted.

Black Body or Full Radiations.—Different substances differ in their radiating and absorbing capacity according to their nature and temperature, and it therefore becomes necessary to have a standard for reference. The standard commonly adopted is what is termed a " black body," and this was defined by Kirchhoff as " A body which is not in the least transparent, and which does not reflect at all, but absorbs all incident radiation of whatever wave-length at every temperature." No substance has been found which fulfils perfectly the condition of a " black body," but it was independently proved by Balfour Stewart and Kirchhoff about the year 1858 that the radiation within an opaque enclosure at a uniform temperature fulfils black body conditions and is independent of the substance forming the enclosure.* The radiations issuing from a small hole in the wall of such an enclosure constitutes " black body " radiations. Therefore, the radiations from a furnace whose walls are at the same temperature as the furnace nearly fulfils black body conditions, as also does the radiations issuing from the bottom of a cavity heated to a uniform temperature.

Kirchhoff's Law.—The absorptive power of a body for radiations of a given wave-length was defined by Kirchhoff to be " the ratio of the radiant energy absorbed to the total incident radiant energy." The absorptive power of a black body according to this definition is unity, since such a body absorbs all incident radiations. If a is the absorptive power and e the emissive power of a body at any temperature, then Kirchhoff's law asserts that " The ratio of the emissive power e to the absorptive power a is the same for all bodies for radiations of the same wave-length and the same temperature, and is equal to the emissive power of a black body.

A body is said to be selective in its radiations when the energy radiated in the various wave-lengths throughout the whole spectrum do not bear a constant proportion to the energy radiated by a black body in the same wave-lengths at the same temperature.

A body is said to be " grey " when it does not radiate as much energy in the various wave-lengths as a black body at the same temperature, but in any wave-length the intensity of the radiation is a constant fraction of that of the black body in the same wave-length. Practically all known substances are more or less selective in their radiations, and it may be said that there are no perfectly " grey " bodies.

Stefan-Boltzmann Law.—From a consideration of some of Tyndall's radiation experiments, Stefan in 1879 suggested that the total radiation emanating from a body is proportional to the fourth power of the absolute temperature. This law has been verified by direct experiment by Lummer and

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* For proof see Preston's *Theory of Heat*, p. 586, 1904 edition.

Pringsheim and by other experimenters for black body radiations, but some experimenters have found considerable deviations in the case of bodies not conforming to black body conditions.

In 1894 Boltzmann furnished a mathematical proof of the Stefan fourth-power law for the case of black body radiations based upon the electromagnetic theory of light.*

Thus, if τ is the absolute temperature of a black body in degrees Cent., the total energy radiated per square centimetre per second is $5{\cdot}32 \times 10^{-5}\tau^4$ ergs. If the black body is surrounded by a black envelope at an absolute temperature τ_0 then the resultant loss of energy by the black body becomes $5{\cdot}32 \times 10^{-5} (\tau^4 - \tau_0^4)$ ergs per sq. cm. per second. If the absolute temperatures are expressed in degrees Fahr. the above expression reduces to $16 \times 10^{-10} (\tau^4 - \tau_0^4)$ British Thermal Units per sq. ft. per hour. Direct calculation shows that even if τ_0 is one-half of τ the influence of τ_0 on the net loss of energy by radiation of the hot body is only about 1 in 16, and for smaller ratios the effect of τ_0 is practically negligible.

Although there is no absolutely black body except as an enclosure under the conditions specified previously on p. 5, it is generally taken that a sooted surface or the surface of a bed of burning coal radiates and absorbs practically as a black body. For ordinary clean brick surfaces, however, the radiating and absorptive powers at high temperatures is probably not more than one-half of the values for a sooted surface, but of course a brick surface would reflect or disperse some portion of the radiations falling on it, whereas a sooted surface has only small reflecting power.

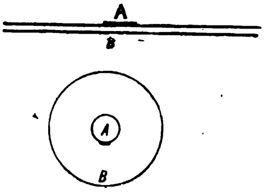

Fig. 1.—Radiations between two plane surfaces and between two cylinders or spheres.

The Stefan-Boltzmann formula given above can only be applied as it stands to the conditions where the hot black surface "sees" nothing but the colder envelope, a condition rarely occurring in practice. It would be fulfilled, for instance, in the circumstances represented in

---

* For the proof refer to Preston's *Theory of Heat*, p. 596, 1904 edition.

Fig. 1 between two infinite plane surfaces and between two cylinders of infinite length or two spheres one inside the other. A is taken to represent the hot body and B the colder body or envelope.

Considering the radiations from a portion of the hot surface, shown thickened in the diagrams, it is evident that the net amount of energy lost by the hot surface to the envelope depends upon the nature, temperature, and area of the hot surface, and is independent of the area or shape of the colder surface, but the proportion of the radiations absorbed by B would depend upon its absorptive power. For the two cases represented in Figs. 2 and 3 the amount of energy emanating from the element of surface A falling on surface B depends upon the solid angle θ.

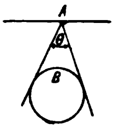

Fig. 2.—Radiations from A within the angle θ.

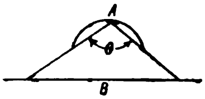

Fig. 3.—Radiations from A within the angle θ.

It should be noted that if heat is being generated at the surface A any reflection and radiation from the colder surface B which is absorbed by the surface A will tend to raise the temperature of A. This explains why a furnace surrounded by firebrick surfaces usually attains to higher temperatures than a furnace surrounded by comparatively cold surfaces.

Of all the conditions met with in industrial practice probably the nearest approach to the conditions to which the Stefan formula on p. 6 can be applied occurs inside a locomotive firebox. In this case the bed of fuel is completely enclosed by a colder surface at nearly a uniform temperature which absorbs heat almost like a "black body" surface. The bed of fuel might be assumed to be a plane surface radiating as a black body, though it is evident by considering an extreme case that the amount of energy radiated will depend upon the area and arrangement of the surface. Thus, for example, a bed of fuel of semicircular shape, indicated by the dotted lines in Fig. 4, would have a larger radiating surface than a flat bed. It is a common practice to build a firebrick arch over a part of the bed of fuel to promote combustion of the hydrocarbon gases and to direct the flow of gases from the firebox into the tubes. Although the arch prevents some of the radiations from the bed of fuel from reaching the relatively cold heating surface, the other side of the arch radiates to the heating surface, though perhaps not as effectively as a black body.

These statements, however, do not represent the whole of the circumstances involved in the radiation inside a locomotive firebox. Usually the firebox is more or less filled with flame containing solid particles of carbon or soot which of itself has certain powers of radiation, absorption, reflection, and transmission. Although the flames extend the radiating surface, as to whether it augments the transmission of radiant heat from the bed of fuel is subject to much uncertainty. Most probably, however, the flames increase the amount of heat radiated, for if the flames do absorb radiant heat from the bed of fuel they probably radiate more than this to the walls. Also, radiations from the bed of fuel absorbed or reflected by the flames tends to raise the temperature of the furnace, as also does the radiations from the flames absorbed by the fuel. Therefore, until more is known of the relative powers of absorption, radiation, reflection, and transmission

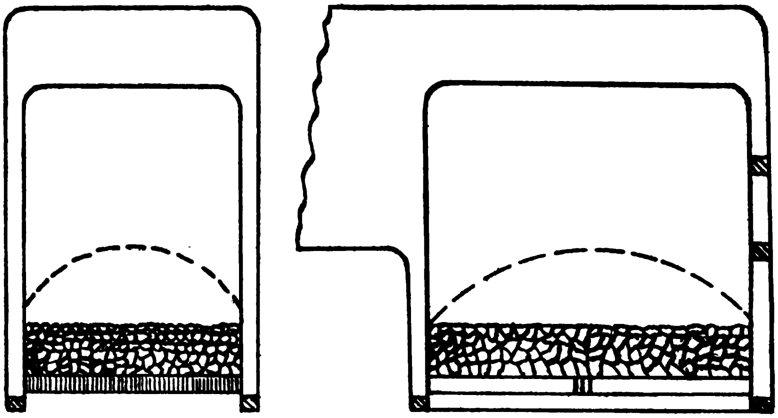

Fig. 4.—Radiations from bed of fuel in a locomotive firebox.

of the flames in the firebox, one cannot say with certainty that the Stefan formula on p. 6 is applicable to the radiations from the bed of fuel when estimating the amount of radiant heat absorbed by the firebox plates.

Taking it, however, that this formula may be applied for approximate calculations, and considering a locomotive firebox of grate area 20 sq. ft. (flat bed of fuel), firebox surface 120 sq. ft. temperature in furnace 2500° F. (2960° F. absolute), temperature of plates 400° F. (860° F. absolute), then,

$$\text{Heat radiated by fuel per hour per sq. ft.}\left\{\begin{aligned}&=1600\left\{\left(\frac{2960}{1000}\right)^4-\left(\frac{860}{1000}\right)^4\right\}\\&=1600\{76{\cdot}7-{\cdot}55\}\\&=122{,}000\text{ B.Th.U.}\end{aligned}\right.$$

On the average,

$$\text{Radiant heat absorbed per hour per sq. ft. of firebox surface}\left\}\begin{aligned}&=\frac{122{,}000\times 20}{120}\\&=20{,}300\text{ B.Th.U.}\end{aligned}\right.$$

The conditions inside the furnace of a Lancashire or a Cornish boiler are much the same as those in the locomotive firebox just discussed, but after leaving the furnace tubes or flues the gases usually pass through brickwork flues into contact with the outside shell of the boiler. Therefore while heat is given to the shell by the contact of the gases the brick surfaces also radiate heat, most of which falls on and is absorbed by the shell plates. The loss of heat by radiation from the brickwork must be compensated by heat taken up from the flowing gases, and therefore the gases will have an average temperature higher than that of the brickwork by an amount necessary to cause the transmission of the heat from the gases to the brickwork.

It is perhaps necessary to examine further the radiating conditions in a shallow cavity, such as that forming the bottom or the side flues of a Lancashire boiler. For the purpose of demonstration suppose a shallow enclosure ABCD of rectangular section to be at a uniform temperature, and having the walls all of the same radiating, absorbing, and reflecting powers. It is evident that the total energy radiating from the cavity ADCB which is absorbed by the wall AB must be equal to that emanating from AB absorbed by the walls ADCB, since the temperature is uniform. Therefore the cavity ADCB is a radiator equivalent to the surface AB, and thus such a cavity would radiate to the shell of a Lancashire boiler practically as a flat surface of area (AB × length). The brick walls would probably be covered with a deposit of soot, coal dust, and light ash, and might be taken to radiate as a black body. No doubt a small portion of the radiant energy would be absorbed by the flue gases, but these would themselves radiate to the walls and to the boiler plates.

The conditions inside the furnace of an ordinary water-tube boiler are not quite the same as those discussed previously in connection with the locomotive firebox. Considering the case of a water-tube boiler having the tubes and fire-grate in the positions shown by Fig. 5, it is evident that if there is no firebrick arch, and with the usual depth of furnace, the greater part of the radiations from the bed of fuel not absorbed by the flames fall directly on the boiler tubes. Most likely the tubes would absorb all these incident radiations, and similarly the bed of fuel would absorb all incident radiations. If we now consider the influence of the firebrick wall AC, it is obvious that the radiations from the bed of fuel within the area ABC which pass through the flames fall on AC. A part of these, say perhaps one-half, would be reflected or dispersed from the brick surface AC, and some of these reflected or dispersed radiations would eventually fall on, and be absorbed by, the

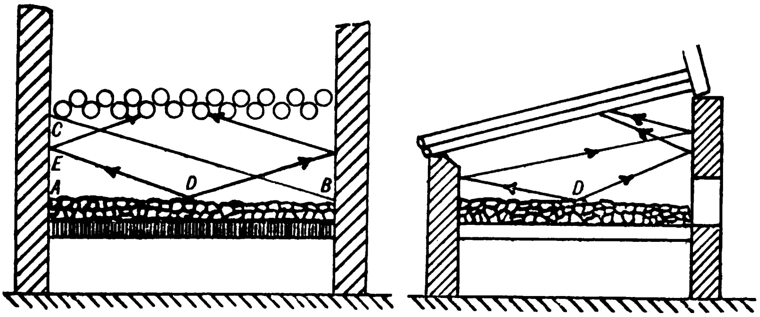

FIG. 5.—Radiations in a furnace having firebrick walls.

tube surfaces. Those radiations not reflected are absorbed by AC and, neglecting the small amount of heat lost by conduction through the brickwork, an equal amount would be radiated, probably in all directions.* From symmetry it may be assumed that about one-half of these would eventually reach the tubes and the other half reach the bed of fuel. Thus, of the total radiant energy from the fuel falling on AC about one-half eventually reaches the tubes and the remainder is reabsorbed by the fuel, this latter portion tending to raise the temperature of the furnace. It would follow, therefore, that much the greater part of the radiation from the fuel ultimately reaches the tubes with ordinary furnace arrangements when there is no firebrick arch. The presence of the flames would modify this argument somewhat, but to what extent is not known, and therefore no useful purpose would be served by discussing such questions as the influence of the depth of the furnace, or the nature and position of the furnace, upon the absorption of the radiant energy from the fuel.

To promote the combustion of bituminous coal in water-tube boilers by raising the temperature of the furnace it is the common practice to build a firebrick arch over the fire-grate. Similarly, when burning gaseous or oil fuel the combustion chamber is usually built as a firebrick enclosure, so that the temperature is as high as is practicable and combustion more or less complete before the gases reach the cold tubes or plates.

Wien's Law for Radiant Energy.—Wien found definite relations between the radiant energy, the wave-length, and the temperature for the radiations from a "black body." When the radiations are dispersed by means of a suitable prism, say of fluor-spar or of sylvin, which substances are transparent for the long heat waves, it is found that there is some particular wave-length for which the intensity of the radiant energy is a maximum. This wave-length of maximum intensity is connected with the temperature of the radiating black body by the two relationships discovered by Wien, and these are usually spoken of as "the displacement laws of Wien." The first is,

---

* It is interesting to note that although the radiations from the fuel bed falling on AC are included within the angle ABC, the nearly equal amount radiated and dispersed by AC would be emitted in all directions.

$$\lambda_{m}\tau=C_1 \quad \ldots\ldots\ldots\ldots \quad (1)$$

where C_1 is a constant, λ_m is the wave-length of maximum radiant energy, and τ is the absolute temperature of the black body. The other is,

$$I_{m}=C_2\tau^5 \quad \ldots\ldots\ldots\ldots \quad (2)$$

where C_2 is a constant and I_m the maximum intensity of the radiant energy.

Lummer and Pringsheim found the value of C_1 to be 2940 for black body radiations, where λ_m is expressed as the number of thousandths of a millimetre and τ the absolute temperature in degrees Cent.

Experiment shows that even with bodies of large reflecting power the value of C_1, although it may vary somewhat with temperature, does not differ greatly from that for a black body. For bright platinum $C_1=2630$, but in this case the total energy of radiation varies nearly as the fifth power of the absolute temperature as compared with the fourth power in the Stefan-Boltzmann expression for a black body.

Wien also proposed an expression of the following form for the intensity of the black body radiations of any given wave-length,

$$I=\beta\lambda^{-5}\epsilon^{-\frac{\beta_1}{\lambda\tau}} \quad \ldots\ldots\ldots\ldots \quad (3)$$

where β and β_1 are constants and ϵ is the base of the natural logarithms, i.e. 2·7183. In order to make the expression fit in more accurately with the experimental results for long wave-lengths Planck proposed to modify the expression as follows :—

$$I=\beta\lambda^{-5}\epsilon^{-\frac{\beta_1}{\lambda\tau}}\left(\frac{1}{1-\epsilon^{-\frac{\beta_1}{\lambda\tau}}}\right) \quad \ldots\ldots\ldots \quad (4)$$

For the short wave-lengths within the visible portion of the spectrum, that is, between ·8μ and ·4μ (μ=·001 mm.) the portion within the brackets is nearly unity and within this range it reduces to Wien's expression (3).

Taking logarithms for Wien's equation,

$$\log_e \mathrm{I}=\log_e \beta-5 \log_e \lambda-\frac{\beta_1}{\lambda\tau} \quad . \; . \; . \; . \; . \; (5)$$

If the intensity of the radiations I_1 and I_2 are measured for a given wave-length at the two absolute temperatures τ_1 and τ_2, then,

$$\log_e \frac{\mathrm{I}_1}{\mathrm{I}_2}=\frac{\beta_1}{\lambda}\left(\frac{1}{\tau_2}-\frac{1}{\tau_1}\right) \quad . \; . \; . \; . \; . \; . \; . \; . \; (6)$$

This expression (6) was discussed in connection with the Wanner optical pyrometer on p. 85 of *The Measurement of Steady and Fluctuating Temperatures*, and was used also on p. 90 of that volume to express the influence of the emissivity on the black body temperature.

Radiation from Gases.—Some experiments have been made by Professor Callendar* on the radiation emitted from the flame of a Mêker burner using coal gas of calorific value 470 to 500 B.Th.U. per cubic foot. The burner had a nickel grid 3 cm. (1·18 in.) diameter, and consumed gas at the rate of ·185 cubic feet per minute. A special radiation pyrometer was used to measure the heat radiated in a particular direction, and assuming the flame to radiate equally in all directions the total radiation was calculated. The gas and air mix before combustion, giving a flame similar to that from a Bunsen burner. With full air supply, in about the proportions required for complete combustion, the burner gives a homogeneous conical pointed flame, but as the air supply is cut down a brilliant inner cone appears. The following results were obtained :—

Total radiation, per cent of heat of combustion	10·5	12·3	14·0	15·9	14·1	14·6	17
Ratio air to gas by volume . . .	5	4	3	2·5	1·5	1	0

It will be noticed that the percentage of heat radiated rises steadily with increase of size of flame from 10·5 with full air to nearly 16% as a maximum with a bright inner cone.

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* *British Association Report*, 1910.

Beyond this point the inner cone becomes ill-defined, the flame flickers, and the radiation falls off to about 14%, rising again to about 17% as the flame becomes luminous when no air is supplied to the burner.

The radiation per unit of surface, or per unit of solid angle subtended, has been termed by Professor Callendar the "intrinsic radiance" R. It is known that a flame is to a certain extent transparent to its own radiations, and in order to determine the manner in which the intrinsic radiance R varied with the thickness x of the flame in the line of sight, and to measure the coefficient of absorption, six precisely similar burners were mounted in a row along the axis of the radiation pyrometer. Several series of measurements were taken with one to six burners lighted in different orders, for two distinct states of the flame which were easily reproducible, viz. (1) with full air supply, and (2) with the inner cones 2·5 cm. high. In the latter case the flames all touched each other, the layer of flame was 21·6 cm. thick and was sensibly homogeneous.

Summary of Observations:

(1) Full air supply; mean thickness per flame 2·8 cm.

Number of flames . .	1	2	3	4	5	6	
Radiation observed, R,	68	124	171	214	250	282	(arbitrary units).

The radiation R can be nearly expressed in terms of the thickness x cm. by the expression, $R=473\ (1-\epsilon^{-\cdot 0537x})$.

This gives in the limit when $x=0, \frac{R}{x}=473\times\cdot 0537=25\cdot 4$ per cm. thickness.

In the limit $x=\infty$, $R=473$.

(2) Inner cones 2·5 cm. high; mean thickness per flame 3·6 cm.

Number of flames . .	1	2	3	4	5	6	
Radiation observed, R,	72	122	165	197	232	261	(arbitrary units).

Formula for $R=373\ (1-\epsilon^{-\cdot 0541x})$.

In the limit when $x=0$, $\frac{R}{x}=373\times\cdot 0541=20\cdot 2$ per cm. thickness.

In the limit $x=\infty$, $R=373$.

It would be recognised, of course, that there was no attempt made to measure the temperature of the flame in these experiments, and it is likely that the temperature of the flame would rise with the number of burners. In any case it is seen from the foregoing results that the radiation from a flame of, say, 2 ft. depth would be practically equal to that from a similar flame of great depth.

Effect of Temperature and Pressure.—The influence of temperature and pressure on the intrinsic radiance of a flame of this character can be predicted theoretically with a reasonable degree of probability but would be difficult to determine experimentally. Within moderate limits of pressure the radiating and absorbing powers of a flame per unit of thickness, and having a given temperature and composition, should vary as the number of molecules present, that is, directly as the density or pressure. Therefore the coefficient of absorption would vary directly as the pressure, and the limit for an infinite thickness would be unaltered. In other words, a layer of gas 0·1 in. thick and 10 atmospheres pressure should radiate at the same rate as a layer 1 in. thick at atmospheric pressure, whilst for an infinite thickness the radiation would be about the same at both pressures.

Before discussing the influence of temperature it is necessary to refer briefly to the nature and origin of the radiation from flames. Experiments made by Julius* show that the radiation from a Bunsen flame is almost wholly due to the CO_2 and steam (H_2O) molecules,† and the *quality* of the radiations depends only on the nature of the burnt and not on that of the burning gases. Also the *quantity* of radiation is dependent only upon the number of CO_2 molecules or the number of steam molecules (H_2O) formed, without regard to the nature of the gases used for combustion.

On examining the radiation from a flame producing both CO_2 and H_2O by using a prism, it is found that most of the radiation is concentrated into two bands, the wave-lengths of which are respectively 4·4μ due to the CO_2 and 2·8μ due to the H_2O, where μ stands for ·001 mm. Taking a mean wave-length of 3·5μ Professor Callendar has compared the intensity of the radiations, using Planck's equation 4, p. 12, with the Stefan fourth power law for the radiation from a black body in order to indicate how the radiation probably varies with the temperature. The results obtained were :—

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* *Die Licht und Wärmestrahlung verbrannter Gase*, Dr. W. H. Julius, Berlin, 1890.

† Professor Tyndal has shown experimentally that CO_2 and H_2O vapour absorb heat radiations. See Preston's *Theory of Heat*.

Absolute temperature, °C. .	1000	1500	2000	2500	3000
Radiation (Planck's equation)	·016	·059	·142	·233	·331
Radiation (Stefan) . .	·009	·045	·142	·347	·721

Thus, according to Planck's law the rate of variation for a single wave-length is much slower than the fourth power law for a black body, and tends to be nearly proportional to the absolute temperature at high temperatures. Probably the actual rate of variation should lie between these limits, but nearer to the rate given by Planck's equation.

The absolute value of the intrinsic radiance from these flames was measured by Professor Callendar by comparison with the radiation of a black body, using the same pyrometer for both. Thus it was found that with six flames with full air supply, having a total thickness 16·8 cm., the intrinsic radiance was the same as that of a black body at 679° C.,* or 952° C. absolute. Calculating from the formula on p. 6, it was found that a black body at 952° C. (1714° F. abs.) radiated 63 cals. per sq. cm. per minute, or about 14,000 B.Th.U. per sq. ft. per hour. The limiting value of the intrinsic radiance for infinite thickness would be 105 cals. per sq. cm. per minute (23,300 B.Th.U. sq. ft. hour) in case (1) of p. 14, with full air supply, and 83 cals. per sq. cm. per minute in case (2).

Taking the flame temperature in case (1) to be about 3300° F. (3760° F. abs.) the radiations from a black body at this temperature would be about 320,000 B.Th.U. per sq. ft. per hour. Therefore the radiation from the Bunsen flame of infinite thickness with full air supply, case (1), is only about $\frac{23,300}{320,000} = \cdot 073$ or 7·3% of that from a black body at the same temperature. If that from a luminous gas flame with ordinary coal gas were taken to be about 1·7 times that of the Bunsen flame, this would make the luminous flame radiation 7·3 × 1·7 = 12·4% roughly of that from a black body.

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* The temperature of the flame itself would probably be as high as 1650° C. (3000° F.).

The radiation from the luminous flames in an ordinary boiler furnace is probably somewhat greater than that from a luminous coal gas flame at the same temperature, because the furnace flames are likely to contain a certain amount of carbon in the form of soot or coal dust, which would make the flame a more effective radiator. It is probable, therefore, that the total radiations reaching the heating surface is greater with luminous flames than with clear flames, but to what extent is at present uncertain.

Radiation of Gas in an Explosion Cylinder.—In the British Association Report for 1910 the late Professor B. Hopkinson gave some results of work carried out by one of his research students, Mr. W. T. David, on a cylindrical cast-iron explosion vessel 30 cm. long and 30 cm. (11·8 in.) diameter. A mixture of 15% of Cambridge coal gas was exploded in the vessel, and series of records were taken of the radiation from the cylinder and of the pressure on a time base. The radiation from the gases were allowed to fall upon a bolometer (radiation pyrometer) after passing through a window of fluorite. This bolometer was of platinum blackened with lampblack, and its rise of temperature was measured on a time base. The inside of the cylinder was lined with polished silver so as to reflect as much as possible of the incident radiations. No allowance was made for the absorption of radiations by the fluorite window, said to amount to 5% of the total. The following values illustrate the character of the results obtained:—

Time from ignition, seconds.	Temperature of gas.		Total heat radiated as percentage of heat of combustion.
	°C.	°F.	
·05	2090	3794	3·0
·10	1870	3398	8·5
·15	1690	3074	12·5
·20	1510	2750	15·5
·30	1290	2354	19·0
·40	1110	2030	21·0
·50	980	1796	22·0

The record of pressures, obtained by means of an optical indicator recording on a travelling photographic film, showed that the maximum pressure was attained about ·05 seconds after ignition. The above table shows that the amount of heat radiated up to that time was about 3% of the heat of combustion. Up to ·5 seconds the total amount radiated was 22%. Thus it is shown that the exploded gases inside a gas-engine cylinder is a fairly effective radiator of heat.

The Influence of Radiation on the Measurement of Gas Temperatures.—Attention was directed on p. 3 to the difficulties associated with the accurate measurement of the temperatures of gases on account of radiation from and to the thermometer, particularly so when the walls of the enclosure are at a lower temperature than the gases. Many instances could be given of boiler tests where the gases leaving an ordinary boiler are recorded as having a temperature very little higher than that of the water in the boiler when working at the normal rate. A critical examination generally shows that the temperature was obtained by a single thermometer placed in a wide flue and subjected to radiations from more or less of the comparatively cool boiler surfaces.

The writer has tried the effect of placing a mercury thermometer in the stream of the gases among the back bank of tubes of a Stirling boiler. The thermometer was enclosed in an iron tube, having a slit on one side opposite the scale, and arranged so that this tube with the thermometer could be rapidly withdrawn and the thermometer read. The metal round the bulb of the thermometer prevented the temperature from falling while the thermometer was being read. It was found that the average recorded temperature over a period of three hours was about 376° F., whilst a platinum-resistance thermometer placed at the flue gas exit and on the boiler side of the damper gave an average temperature of about 500° F., although even in this thermometer the sheath was exposed on one side to the radiations from the bottom boiler drum. The temperature of the feed water entering the top back drum was about 230° F. and the steam temperature was 355° F. It is fairly certain that the boiler tubes near the thermometer would not be far from 300° F.

Another particular example of this defective estimate of gas temperature is given in Bulletin 23 of the Bureau of Mines, Washington, on "The Steaming Tests of Coals and Related Investigations" and with respect to a test on a Stirling boiler having air-heater tubes near the flue gas outlet. The bulb of the thermometer was about 1 ft. from the air-heater coils, midway between the back wall and the back steam drum, and about 3 ft. below the damper which was at the gas outlet at the top of the boiler. The temperature of the back drum was probably below 352° F., the temperature of steam corresponding to about 125 lb. per sq. in. gauge pressure, and most likely the air-heater tubes would be below this temperature. The thermometer received heat from the gases scrubbing against it and would radiate heat to the comparatively cold surrounding surfaces. The thermometer indicated a temperature considerably below that of the gases, as was discovered by placing another thermometer in the hood of the stack about 4 ft. above the damper. The hood of the stack extended over the uptake of another boiler which formed one battery with the boiler under test, but during the test this other boiler was banked and the flue gases, which were at a very low temperature, entered the same hood as the gases from the test boiler. There was no partition separating the two halves so that the cool gases from the banked boiler could partly mix with those from the test boiler and lower the temperature of the resultant mixture. Besides this, the sheet iron of the hood always radiated heat and was therefore colder than the gases inside. Notwithstanding these influences the thermometer in the hood indicated a temperature about 100° F. higher than the temperature shown by the thermometer below the damper, and without doubt this difference was due to the close proximity of the cold air-heater tubes and the rear steam drum. The thermometers were exchanged several times, but their indications always bore the same relation no matter what thermometer was used. There could be no question of after-burning of the gases above the damper as the recorded temperatures were of the order 373° F. below and 470° F. above. Such an example illustrates the futility of calibrating a flue-gas thermometer for accuracy, and then to place the thermometer in a position where it cannot possibly indicate the true gas temperatures. The above considerations led the experimenters* to place the flue gas thermometer in a special shield in the uptake of the Heine boiler they were afterwards testing, which they also used as a gas sampler.

An extensive series of experiments† on different boilers have been made by H. Kreisinger and J. F. Barkley on the influence of radiation in measuring the temperature of boiler flue gases. Two series of hot junctions were compared against each other at different points in each boiler, one junction made of wires ·008 in. diameter and the other made as a tube ·5 in. diameter. By extrapolation the experimenters also estimated the true gas temperatures. The following values illustrate the character of the results obtained:—

Type of Boiler.	Position of Hot Junctions.	Average reading by fine wire junction °F.	Average reading by thick junction °F.	Estimated true temperature °F.	Error. By fine wire junction °F. Col. 5–Col. 3.	Error. By thick junction °F. Col. 5–Col. 4.
1	2	3	4	5	6	7
Babcock and Wilcox water tube. Gas fired.	Below tubes but exposed to radiation from fire-brick.	2050	1730	2160	110	430
	Among tubes .	1610	1210	1780	170	570
	,, ,, .	536	500	554	18	54
Stirling water tube. Gas fired.	One foot above arch and one foot in front of first bank of tubes.	1980	1600	2080	100	480
	Among tubes .	1880	1484	2030	150	546
	,, ,, .	1260	1026	1350	90	324
	,, ,, .	680	608	710	30	102

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* Messrs. L. P. Breckenridge, H. Kreisinger and W. T. Ray.

† "Radiation Error in Measuring Temperature of Gases," *Journal Amer. Soc. Mech. Engs.*, May, 1917.

The values in column 7 indicate what a large error may be incurred by using a pyrometer enclosed in a tube of ½ in. diameter if not shielded from the radiations emitted by the relatively cold tube surface.

Such an arrangement as that referred to on p. 20 for screening a thermometer bulb or tube is not suitable for use in the furnace or combustion chamber of a boiler. For this purpose it is sufficient to arrange a small screen of asbestos or refractory material to surround the bulb without unduly preventing the gases from coming into thorough contact with the bulb, or the equivalent of the bulb in electrical pyrometers.

CHAPTER II

CONDUCTION OF HEAT

WHEN one end of a metal bar or rod is heated a flow of heat occurs from the hot end towards the colder end and this heat is said to be conducted along the bar. In this conduction of heat there is no apparent transference of the particles or molecules themselves. According to our accepted ideas of the molecular construction of matter, the molecules are more or less independent of one another and can only exert an influence on one another either through the all-pervading ether or by impinging on or jostling one another. In the case of radiation the radiating body and the receiving body are so far apart as to preclude direct contact, but with conduction there may be a certain amount of direct contact between neighbouring molecules. In recent years the theory has been enunciated that conduction of heat is associated to some extent with the transference of very small particles or atoms, termed "electrons," which carry charges of electricity as well as heat. Quite apart from such theories, however, the laws of conduction have been determined by experiment.

Consider first the conduction of heat through a plate of metal where the thickness of the plate is small compared with the radius of curvature. Assume that the temperature over a large portion of the hot surface is θ_1 and that at the other side is θ_2, A the area of the surface, and q the thickness of the plate. It is well established by experiment that the amount of heat H_0 flowing through the plate in the time t can be expressed by,

$$H_0 = k\frac{(\theta_1-\theta_2)}{q}At \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (1)$$

where k is the conductivity, practically independent of the temperature of the plate for the comparatively small difference $(\theta_1 - \theta_2)$ common in boilers and condensers.

The steel tubes of a locomotive boiler are usually about ·1 in. thick, which is small compared with the usual diameter (about 2 in.) of the tubes. If 6000 B.Th.U. per hour pass through the metal per square foot of heated surface and the conductivity is taken at ·01 B.Th.U. per sq. ft. per sec. per deg. Fahr. difference of temperature per foot thickness, then the difference of temperature $(\theta_1 - \theta_2)$ is calculable thus,

$$6000 = \cdot 01 \times \frac{(\theta_1 - \theta_2)}{\frac{\cdot 1}{12}} \times 1 \times 60 \times 60,$$

$$\text{or,}\quad (\theta_1 - \theta_2) = \frac{6000 \times \cdot 1}{\cdot 01 \times 60 \times 60 \times 12} = 1\cdot 4\ ^\circ\text{F. nearly} \quad . \quad . \quad . \quad . \quad (2)$$

Even for a plane plate of mild steel ½ in. thick conducting the same amount of heat the difference of temperature between the two sides would only be about $5 \times 1\cdot 4 = 7^\circ$ Fahr.

If the conductivity of copper is taken to be about six times that of mild steel, then for the above-mentioned transmission of heat it follows that the difference of temperature between the two sides would only be about ⅙th that for steel with the same thickness of plate.

Considering the case of a boiler where the average difference of temperature between the gases and the water may be something like 1000° F., it is evident that whether the plate is of steel or of copper it can hardly influence the rate of heat transmission appreciably seeing that the drop of temperature through the metal in either case is only a very small portion of the total difference between the gas and the water. With condensers and similar apparatus, however, the circumstances are different because the difference of temperature between the steam and the water is usually only a few degrees when transmitting the same amount of heat per unit of area as a boiler tube.

For a circular tube which is transmitting heat at a constant rate the drop of temperature through the metal of the tube is not correctly expressed by the preceding calculations. Assume the temperature conditions along the length l of the tube are constant, and let r_1 and r_2 be the internal and external radii respectively. If H_1 units of heat are transmitted per unit of time and the gradient of temperature across a section of the tube at the radius r is $\frac{d\theta}{dr}$, then,

$$H_1 = -k2\pi rl\frac{d\theta}{dr},$$

$$\text{or,}\quad \theta_2-\theta_1=\int_{r_1}^{r_2} d\theta = -\frac{H_1}{k2\pi l}\int_{r_1}^{r_2}\frac{dr}{r}$$

$$= -\frac{H_1}{2k\pi l}\log_e\frac{r_2}{r_1}.$$

But if H units of heat are transmitted per unit area of inner surface,

$$H=\frac{H_1}{2\pi r_1 l}$$

$$\text{and thus, } \theta_2-\theta_1=-\frac{Hr_1}{k}\log_e\frac{r_2}{r_1} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; (3)$$

For example, if $r_2=1$ in., $r_1=\cdot 9$ in., that is, thickness of metal ·1 in., and heat transmission is again 6000 B.Th.U. per hour per sq. ft. of inner surface, then for a mild steel tube,

$$\theta_2-\theta_1=-\frac{6000\times\cdot 9}{60\times 60\times\cdot 01}\log_e\frac{1}{\cdot 9}$$

$$=-1{\cdot}58^\circ\ \text{F}.$$

comparable with the value $-1{\cdot}4^\circ$ F. as obtained by equation 2, p. 23. It is seen from this calculation, therefore, that equations 2 and 3 lead to practically the same result unless the thickness of the tube is large compared with the diameter.

Equations 2 and 3, however, only refer to the simplest conditions of the conduction of heat, and therefore the subject will now be treated more generally.

General Equations for Conduction of Heat.—Consider the flow of heat along a bar of uniform section heated at one end at a constant temperature and losing heat from the surface of the bar at a constant rate to the atmosphere. For some time after the heat is applied at the end the temperature of the bar will gradually rise, but eventually each point would acquire a steady temperature and then a steady flow of heat would take place along the bar. Those sections nearest the heated end would acquire the steady condition first, and the points further and further remote from the heated end would acquire the steady state in succession. Assume that the steady temperature has been attained at all points along the length of the bar. Referring to Fig. 6 consider any section of area A at the distance x from the heated end where the temperature is θ above the atmosphere. Let the gradient of temperature be $\frac{d\theta}{dx}$, then the flow of heat H_x per unit of time is given by,

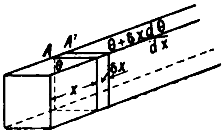

FIG. 6.—Conduction along a bar.

$$H_x = -k\,A\,\frac{d\theta}{dx}.$$

The temperature of an adjacent section at $x + \delta x$ is $\theta + \frac{d\theta}{dx}\delta x$ above the atmosphere and the flow of heat $H_{x+\delta x}$ at this section will be,

$$\begin{aligned} H_{x+\delta x} &= -k_1\,A_1\,\frac{d}{dx}\left(\theta + \frac{d\theta}{dx}\,\delta x\right) \\ &= -k_1\,A_1\left(\frac{d\theta}{dx} + \frac{d^2\theta}{dx^2}\,\delta x\right). \end{aligned}$$

The loss of heat between the two sections from the surface of the bar will be, if $k = k_1$ and $A = A_1$,

$$H_{x+\delta x} - H_x = -kA\frac{d^2\theta}{dx^2}\delta x \quad \ldots\ldots \quad (4)$$

Let the perimeter of the bar $= p$

and surface emissivity $= E$,

Then $$H_{x+\delta x} - H_x = -Ep\theta\,\delta x.$$

$$\therefore\ kA\frac{d^2\theta}{dx^2} = Ep\theta \quad \ldots\ldots\ldots \quad (5)$$

$$\text{or, } \frac{d^2\theta}{dx^2} = u^2\theta, \text{ where } u^2 = \frac{Ep}{kA}.$$

The solution of this equation is,

$$\theta = a\epsilon^{ux} + b\epsilon^{-ux}.$$

If $\theta = o$ when $x = \infty$, and $\theta = \theta_0$ when $x = o$,

Then, $\epsilon^{u\infty} = \infty$, and, $a = o$.

Also, $b\epsilon^{-uo} = b = \theta_0$,

$$\therefore\ \theta = \theta_0 \epsilon^{-ux}.$$

If the bar could be covered with a perfect non-conductor, or if the area of the section were considered to be infinitely great and at a uniform temperature, there would be no loss of heat externally, and if the hot side or end were kept at the constant temperature θ_0 the whole of the bar would ultimately arrive at the same temperature. Therefore the flow of heat through any particular section and at any particular time would become stored in the bar at the more remote portions.

Without specifying a steady temperature at the heated end and the attainment of the steady state, consider the general case of the flow of heat, taking account of the heat capacity of the material of uniform section. Let s be the specific heat of the material and ρ the density, then equation 4 becomes,

$$\mathrm{H}_x - \mathrm{H}_{x+\delta x} = \mathrm{E}p\theta\delta x + \mathrm{A}\rho s\frac{d\theta}{dt}\delta x,$$

where t is the time, and instead of equation 5 we have,

$$k\mathrm{A}\frac{d^2\theta}{dx^2} = \mathrm{E}p\theta + \mathrm{A}\rho s\frac{d\theta}{dt}.$$

If no heat is lost from the bar, then $\mathrm{E} = o$

$$\text{and } k\frac{d^2\theta}{dx^2} = \rho s\frac{d\theta}{dt},$$

$$\text{or, } c\frac{d^2\theta}{dx^2} = \frac{d\theta}{dt} \quad \ldots\ldots\ldots\ldots\ldots \quad (6)$$

where $c = \frac{k}{s\rho}$ (sometimes called the "diffusivity" of the material).

In the case of a steam-engine cylinder which is well covered with non-conducting material and without steam jackets it can be assumed that the inner surface of the wall is subject to a simple harmonic variation of temperature, which, though perhaps not quite true in most cases, is sufficiently near the truth to make the solution of equation 6 of interest and of importance in the study of conduction.

Suppose the average temperature to be of value ϕ, and that the temperature at the inner surface is $(\phi+a \sin \omega t)$. A general solution of equation 6 is then given by,

$\theta-\phi=\mathrm{A}\epsilon^{ax} \sin (\omega t+ax)+\mathrm{B}\epsilon^{-ax} \sin (\omega t-ax)$ where A and B are constants, and $a=\sqrt{\frac{\omega ps}{2k}}$.

Now if $(\theta-\phi)=o$ when $x=\infty$, obviously $\mathrm{A}=o$,
and if $(\theta-\phi)=a \sin \omega t$ when $x=o$, then $\mathrm{B}=a$.
$\therefore\ \theta-\phi=a\epsilon^{-ax} \sin (\omega t-ax)$.

If the engine makes n revolutions per second,
then $\omega=2\pi n$
and $(\theta-\phi)=a\epsilon^{-ax} \sin (2\pi nt-ax)$ . . . . (7)

The above solution refers to the temperature conditions when the mean temperature is the same at all points in the metal. If the outer and inner walls are kept at the mean temperatures ϕ_0 and ϕ respectively, and the thickness of the wall is q, then,

$$\theta=a\epsilon^{-ax} \sin (\omega t-ax)+\phi+\frac{\phi_0-\phi}{q}x \quad . \ . \ . \ . \quad (8)$$

In the case of a steam-jacketed cylinder $(\phi_0-\phi)$ would be positive, whereas with a water-jacketed gas-engine cylinder the value would be negative.

If the above equations are compared with the surface conditions, where $\theta-\phi=a \sin \omega t$, it is seen that the temperature range is reduced from $2a$ at the surface to $2a\epsilon^{-ax}$ at the depth x, and that the temperature phase is caused to lag by the angle ax.

Since $\sin \omega t$ and $\sin (\omega t-2\pi)$ have the same value, then when $ax=2\pi$ the temperature wave lags behind by a complete period, agreeing in phase with that at the surface.

If the surface considered is curved instead of being plane it is assumed in the above solutions that the thickness of the metal is small compared with the radius of curvature.

Fig. 7 illustrates the fluctuations of temperature at various depths for a simple harmonic surface variation of $2a=10°$ F. at the speed of 120 cycles per minute, and Fig. 8 has been drawn to show the range of temperature at various depths. It will be seen that at this speed there is little or no variation of temperature at depths greater than about ¼ in. The following calculation illustrates the method of computation, making use of equation 7, p. 27.

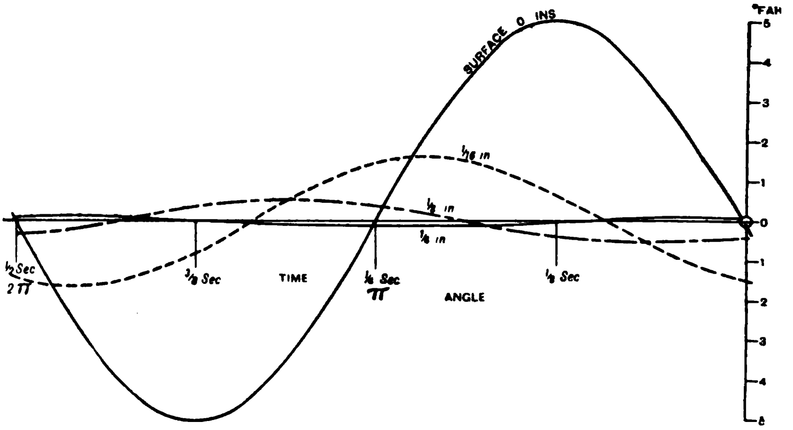

Fig. 7.—Fluctuations of temperature at different depths in wall of engine cylinder.

$$\omega=\frac{2\ \pi\ 120}{60}=4\pi \text{ radians per second.}$$

$$a=\sqrt{\frac{\omega\rho s}{2k}}=\sqrt{\frac{4\pi\times 470\times{\cdot}12}{2\times{\cdot}0075}}$$

$$=217 \text{ ft., lb., sec., units.}$$

At the depth $\frac{1}{16}$ in., or, ·0625 in.

$$\epsilon^{-ax}=\epsilon^{-217\times\frac{\cdot 0625}{12}}$$

$$={\cdot}323.$$

∴ Range of temperature at depth $\frac{1}{16}$ in. $=10\times{\cdot}323$

$$=3{\cdot}23^\circ \text{ F.}$$

Phase lag behind surface $=ax=1{\cdot}13$ radians

$$=64{\cdot}7^\circ$$

$$\text{or, time lag}=\frac{1{\cdot}13}{2\pi}\times\tfrac{1}{2}={\cdot}09 \text{ seconds.}$$

Considering further the conditions of a uniform mean temperature throughout the whole metal, it is evident that there will be an alternate flow of heat into and out from the metal since the temperature of the surface varies during the cycle.

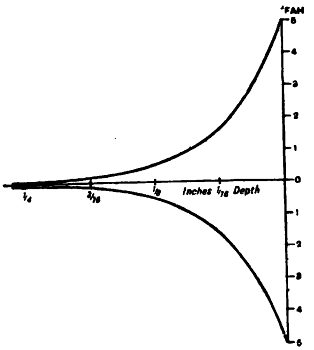

Fig. 8.—Range of temperature fluctuations with change of depth in metal.

Let H=heat-flow per unit time per unit area of surface at any instant.

Then, $H=-k\dfrac{d\theta}{dx}$, where $\dfrac{d\theta}{dx}$ is the temperature gradient at the surface.

From equation 7, p. 27.

$$\frac{d\theta}{dx}=\frac{d}{dx}\left\{ a\epsilon^{-ax}\sin(\omega t-ax)\right\}$$

$$=-aa\epsilon^{-ax}\sin(\omega t-ax)+(-a)(a\epsilon^{-ax})\cos(\omega t-ax)$$

$$=-aa\epsilon^{-ax}\left\{\sin(\omega t-ax)+\cos(\omega t-ax)\right\}$$

Putting $x=o$,

Then, $\dfrac{d\theta}{dx}(x=o)=-aa(\sin\omega t+\cos\omega t)$

or, $H=kaa(\sin\omega t+\cos\omega t)$.

Now, H is zero when $(\sin\omega t+\cos\omega t)=o$.

that is, when $\omega t=-\frac{\pi}{4}$ and $\frac{3}{4}\pi$, and the heat is flowing into the metal during the period between these angles.

Thus, Heat flow in time $\delta t=-k\dfrac{d\theta}{dx}\delta t$.

$$=-k\frac{d\theta}{dx}\,\frac{\delta(\omega t)}{\omega}\quad\ldots\ldots\ldots(9)$$

where ω is constant angular velocity.

Therefore, the amount of heat flowing into the surface per cycle,

$$=\int_{-\frac{\pi}{4}}^{\frac{3}{4}\pi}\frac{kaa}{\omega}\left(\sin\omega t+\cos\omega t\right)d(\omega t)$$

$$=\frac{kaa}{\omega}\Big[-\cos\omega t+\sin\omega t\Big]_{-\frac{\pi}{4}}^{\frac{3}{4}\pi}$$

$$=\frac{kaa}{\omega}\left[\left\{\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}\right\}-\left\{-\frac{1}{\sqrt{2}}-\frac{1}{\sqrt{2}}\right\}\right]=2\sqrt{2}\frac{kaa}{\omega}$$

Using the same values for k and α given or derived on p. 28,

$$k = \cdot 0075, \quad \alpha = \sqrt{\frac{\omega \rho s}{2k}} = \sqrt{\frac{470 \times \cdot 12}{2 \times \cdot 0075}}\,\sqrt{\omega},$$
$$= 61 \cdot 3 \sqrt{\omega}.$$

$$\therefore \frac{2\sqrt{2} k \alpha a}{\omega} = 2\sqrt{2} \times \cdot 0075 \times 61 \cdot 3 \frac{a}{\sqrt{\omega}}$$
$$= \frac{1 \cdot 3a}{\sqrt{\omega}} = \frac{\cdot 52a}{\sqrt{n}} = \frac{4a}{\sqrt{\mathrm{N}}} \left\{ \begin{array}{l} \text{B.Th.U. per sq.} \\ \text{ft. per cycle} \end{array} \right. \quad . \quad . \quad . \quad (10)$$

where n and N are the revolutions per second and per minute respectively, and a is one-half the total range of temperature at the inner surface in deg. Fahr. Professor Mellanby * has used this last expression to estimate the range of temperature $2a$ at the inner surface of a steam engine cylinder which would be required if the heat flow is to account for the whole of the " missing quantity " when using saturated steam. He found that no possible range of temperature could account for the whole of the " missing quantity " and concluded that valve leakage caused a portion of the losses.

For the case represented by equation 8, p. 27,

$$\frac{d\theta}{dx}_{(x=o)} = -\alpha a\,(\sin \omega t + \cos \omega t) + \frac{\phi_0 - \phi}{q}$$

This is zero when $\alpha a\,(\sin \omega t + \cos \omega t) = \frac{\phi_0 - \phi}{q}$

Solving this would give the angular duration of the heat flow into the surface of the metal, and then integrating between these angular limits the amount of the heat-flow per cycle would be calculable.

The amount of heat represented by equation 10, is, of course, temporarily stored in the metal of the cylinder. It is possible to calculate the increased amount of heat actually stored during this period in the metal between the inner surface and any depth x by integrating equation 9 between the angular limits $-\frac{\pi}{4}$ and $\frac{3}{4}\pi$. That is, heat passing through metal at a depth x between $-\frac{\pi}{4}$ and $\frac{3}{4}\pi$,

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* " The Effects of Steam-Jacketing upon the Efficiency of a Horizontal Compound Engine," *Proc. Inst. Mech. Engs.*, June, 1905.

$$\int_{-\frac{\pi}{4}\omega}^{\frac{3}{4}\pi} \frac{k}{\omega}\, a\mathfrak{a}\,\epsilon^{-ax} \left\{\sin(\omega t - ax) + \cos(\omega t - ax)\right\} d(\omega t).$$

$$= \frac{ka\mathfrak{a}\epsilon^{ax}}{\omega}\Big[-\cos(\omega t - ax) + \sin(\omega t - ax)\Big]_{-\frac{\pi}{4}}^{\frac{3}{4}\pi}$$

$$= \frac{ka\mathfrak{a}\epsilon^{-ax}}{\omega}\Big\{-\cos(\tfrac{3}{4}\pi - ax) + \cos(-\tfrac{\pi}{4} - ax)$$

$$+\sin(\tfrac{3}{4}\pi - ax) - \sin(-\tfrac{\pi}{4} - ax)\Big\}$$

For example, consider the results obtained at 120 cycles per minute at depth $x = \cdot 02$ in. $= \frac{\cdot 02}{12}$ ft. in cast iron.

$$a = \sqrt{\frac{\omega\rho s}{2k}} = 217, \qquad \epsilon^{-ax} = \cdot 696.$$

$$\frac{ka\epsilon^{-ax}a}{\omega} = \cdot 09a$$

and, $\{-\cos(\tfrac{3}{4}\pi - \cdot 362) + \cos(-\tfrac{\pi}{4} - \cdot 362) + \sin(\tfrac{3}{4}\pi - \cdot 362)$
$-\sin(-\tfrac{\pi}{4} - \cdot 362)\} = 2\cdot 644.$

∴ Heat flow through section at depth ·02 in. becomes,

$$\cdot 09 \times 2\cdot 644\, a = \cdot 24a.$$

But, heat entering surface in same period $= \frac{4a}{\sqrt{N}} = \cdot 354a$

∴ Heat stored in metal up to depth ·02 in. $= \cdot 354a - \cdot 24a$
$= \cdot 114a.$

It is to be noted that this is about one-third of the total heat entering the surface.

This calculation shows that when a thermo-couple is introduced into a hole bored to within ·02 in. of the inner surface of an engine cylinder to measure the fluctuations of temperature of the metal, the wire or wires forming the junction should be soldered to the metal at the bottom of the hole and should be of relatively large diameter so as to offer a path for the heat flow similar to that in the body of metal.

The expressions so far deduced have reference to the flow of heat in one direction only. The general case, where the flow may take place in any direction, can be deduced by similar methods. Let the three mutually rectangular axes OX, OY, OZ, Fig. 9, be the axes of reference. Consider the flow of heat through a very small rectangular parallelopiped having the edges of lengths δx, δy, δz, parallel to the respective axes. If θ is the temperature of the central point of the parallelopiped, the temperatures of the two faces at right angles to the axis of x will be,

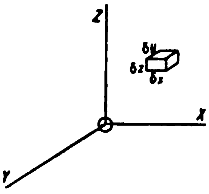

FIG. 9.—Conduction of heat.

$$\theta+\tfrac{1}{2}\frac{\delta\theta}{\delta x}\delta x, \text{ and } \theta-\tfrac{1}{2}\frac{\delta\theta}{\delta x}\delta x$$

where $\frac{\delta\theta}{\delta x}$ is the temperature gradient at the point considered parallel to the axis of x.

Also, the heat-flow at the one face will be,

$$-k\delta y\ \delta z\frac{\delta}{\delta x}\left(\theta+\tfrac{1}{2}\frac{\delta\theta}{\delta x}\delta x\right)$$

and at the other face,

$$-k\delta y\ \delta z\frac{\delta}{\delta x}\left(\theta-\tfrac{1}{2}\frac{\delta\theta}{\delta x}\delta x\right)$$

It follows that the difference between these will be stored in the parallelopiped of volume $\delta x\times\delta y\times\delta z$ and this is,

$$k\frac{\delta^2\theta}{\delta x^2}\delta x\ \delta y\ \delta z.$$

By similar reasoning the total rate of accumulation of heat in the parallelopiped is,

$$k\frac{\delta^2\theta}{\delta x^2}\delta x\ \delta y\ \delta z+k\frac{\delta^2\theta}{\delta y^2}\delta x\ \delta y\ \delta z+k\frac{\delta^2\theta}{\delta z^2}\delta x\ \delta y\ \delta z=\rho s\frac{d\theta}{dt}\delta x\ \delta y\ \delta z,$$

$$\text{or, } k\left\{\frac{\delta^2\theta}{\delta x^2}+\frac{\delta^2\theta}{\delta y^2}+\frac{\delta^2\theta}{\delta z^2}\right\}=\rho s\frac{d\theta}{dt}. \quad (11)$$

If the temperature conditions have become steady at every point, then $\frac{d\theta}{dt}=o$, and the equation reduces to,

$$\frac{\delta^2\theta}{\delta x^2}+\frac{\delta^2\theta}{\delta y^2}+\frac{\delta^2\theta}{\delta z^2}=o. \quad (12)$$

This equation is solved by

$$\begin{aligned}\theta &= m\,(x^2+y^2-2z^2)\\ &= m\,(r^2-2z^2)\end{aligned} \quad \ldots\ldots\ldots \quad (13)$$

where r is the radius measured from the axis of z and θ is the temperature above that at the zero point.

This solution is particularly applicable to the conditions at the end of an ordinary gas or oil engine piston without water cooling of the piston. Here the circular face of the piston receives heat at what may be considered to be a uniform rate,* and this heat has to be dissipated mostly to the cylinder liner and jacket by conduction radially through the end to the cylindrical surface of the piston. Taking the origin at the centre of the piston end on the outside surface (i.e. air-cooled side) and the axis of z to be in the axis of the piston, then the above solution fulfils the conditions of $\frac{\delta\theta}{\delta z}=o$ over the plane $z=o$ and $\frac{\delta\theta}{\delta z}=-4mq$ over the plane $z=q$, where q is the thickness of the end. The equation may not quite satisfy the conditions at the cylindrical surface of the piston where $r=r_1$, the radius of the piston, for here $\theta=m\,(r_1{}^2-2z^2)$. But for pistons of ordinary dimensions the solution is probably sufficiently near to the true conditions for the purposes of calculation.

Thus, at the inner (gas) surface,

$$\mathrm{H}=-k\frac{\delta\theta}{\delta z}=4kmq \text{ per unit of area.}$$

$$\therefore\ m=\frac{\mathrm{H}}{4kq}$$

and, $$\theta=\frac{\mathrm{H}}{4kq}(r^2-2z^2) \quad \ldots\ldots\ldots\ldots\ldots \quad (14)$$

Therefore, the difference of temperature θ_r between the centre and the edge of the piston at the back end is approximately $\frac{\mathrm{H}r_1{}^2}{4kq}$ when q is not large compared with r_1. It should also be noted that the difference of temperature is greater the smaller the thickness q of the piston end, all other things equal, which would suggest that a thin piston end of this type is more likely to cause pre-ignitions than one having an end of thicker metal. As would be expected, the greater the conductivity of the metal the smaller is the value of θ_r. In other words, for given values of H and r_1 the product kq would have a constant value to obtain some predetermined value of θ_r.

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* The metal near the gas-side surface has a small fluctuation of temperature due to the varying gas temperatures, and some heat flows into the metal and from the metal to the gases alternately, but this fluctuation is not important in the present discussion.

Considering a flat-end piston 12 in. diameter and thickness 1·5 in. and supposing that it is receiving heat at the rate of 250 B.Th.Us. per square foot per minute over and above the amount lost to the atmosphere on the other side, then the difference of temperature between the centre and the edge would be roughly,

$$\frac{250}{60} \times \cdot5^2 \times \frac{1}{4 \times \cdot0075 \times \frac{1\cdot5}{12}} = 278^\circ \text{ F.}$$

The late Professor Hopkinson * verified the above approximate law by measuring the actual piston temperatures at three points, using nickel-wrought iron thermo-couples,† and he found it to be reasonably correct. He also covered the air side of the piston end with non-conducting material (magnesia) about 2 in. thick to see whether this had any large effect on the temperatures, and thereby found that there was apparently but little exchange of heat between the bare piston and the atmosphere, seeing that the non-conductor had only a small influence on the temperature of the piston.

A certain drop of temperature is also required between the edge of the piston at the back end and the jacket water, for nearly all the heat which flows through the piston has to be transmitted to the jacket water through the cylindrical portion of the piston, then through the oil film to the cylinder liner, through the liner and from the liner to the water. This drop of temperature can best be studied by considering the following results deduced from Professor Hopkinson's paper, which refer to conditions where there was an explosion nearly every cycle.

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* "Heat-flow and Temperature Distribution in the Gas Engine," *Proc. Inst. Civil Engs.*, Vol. CLXXVI, 1908–09, Part II.

† See *The Measurement of Steady and Fluctuating Temperatures*, p. 115, for description of thermo-couples.

The engine was built by Messrs. Crossley Bros. and worked on the Otto cycle, using Cambridge coal gas. The cylinder diameter was 11½ in. and the stroke 21 in. and the normal speed 180 revolutions per minute. At this speed the maximum brake horse-power was about 40. The compression pressure was higher than is usual, being about 160 lbs. per sq. in. The governing was by the hit-and-miss method and the ignition was by magneto. The area of the piston was ·72 sq. ft. and the total surface exposed to the gases at the in-centre about 2·5 sq. ft.

Let H_t =total heat to jacket water and to external radiation, B.Th.U. per minute.

θ_c =temperature at centre of piston, above mean jacket temperature, °C.

θ_A =temperature of piston 3·25 inches from centre, above mean jacket temperature, °C.

θ_B =temperature of piston 3·62 inches from centre, above mean jacket temperature, °C.

θ_r =calculated temperature of circumference at end of piston, above mean jacket temperature, °C.

H =estimated heat-flow to piston end from gases, B.Th.U. per sq. ft. per minute.

It is assumed that,

$$(\theta_c-\theta_r)=(\theta_c-\theta_A)\,\frac{5{\cdot}75^2}{3{\cdot}25^2}=(\theta_c-\theta_B)\,\frac{5{\cdot}75^2}{3{\cdot}62^2}.$$

The mean jacket temperature was kept at about 42° C. (107·6° F.) during the experiments and it was estimated that the loss of heat by radiation and conduction from the engine (included in H_t) was about 150 B.Th.U. per minute. The table on p. 36 gives some of the values obtained.

A comparison of values of $\frac{\theta_c}{H_t}$ and $\frac{\theta_r}{H_t}$ shows that these are sensibly constant for all values of H_t within the limits of the experiments.

These results, then, on the whole indicate that the temperature of the piston above the mean jacket temperature was about proportional to the total flow of heat to the jacket.

The values of H were calculated from the measured temperatures $(\theta_c-\theta_A)$ and $(\theta_c-\theta_B)$, using the formula on p. 33, and taking into account the variation of the conductivity of cast iron with temperature according to Callendar and Nicolson's values given in Table 24, p. 226. The area of the piston end was ·72 sq. ft. and the values of $\frac{\cdot 72\,\mathrm{H}}{\mathrm{H}_t}$ indicate that the flow of heat to the piston, ·72 H, was roughly proportional to the flow of heat to the jacket, H_t, the average value being about ·12 or 12%.

B.Th.U. per min. to jacket and external radiation H_t	Temperature of piston, °C. (measured above mean jacket temperature, 42 °C).							$\frac{\theta_r}{\theta_c}$	$\frac{\theta_c}{\mathrm{H}_t}$	$\frac{\theta_r}{\mathrm{H}_t}$	Calculated B.Th.U. per sq. ft. per min. to piston H	$\frac{\cdot 72\,\mathrm{H}}{\mathrm{H}_t}$
	θ_c	θ_A	θ_B	$\theta_c-\theta_A$	$\theta_c-\theta_B$	$\theta_c-\theta_r$ calculated.	θ_r calculated.					
1333	270	224	—	46	—	144	126	·467	·202	·0944	225	·121
1968	390	323	—	67	—	210	180	·463	·198	·0916	287	·105
1322	276	227	—	49	—	153	123	·446	·209	·0930	239	·130
1980	419	350	—	69	—	216	203	·485	·212	·1025	278	·101
1327	285	—	224	—	61	154	131	·460	·215	·0988	237	·129
1990	425	—	331	—	94	237	188	·442	·214	·0943	310	·112
2103	437	—	340	—	97	245	192	·439	·208	·0913	316	·108
1912	383	—	289	—	89	225	158	·414	·200	·0830	316	·119
1397*	295	246	—	49	—	153	142	·482	·211	·1017	233	·12
1945*	423	346	—	77	—	241	182	·430	·218	·0935	318	·118

The two last tests were made with the air side of the piston lagged with magnesia about 2 in. thick, but evidently the lagging had little or no influence on the piston temperature or on the ratio $\frac{\cdot 72\mathrm{H}}{\mathrm{H}_t}$.

The expression $\theta_c-\theta_r=\frac{\mathrm{H}r_1^2}{4kq}$ deduced on p. 33 may be used to give an approximate estimate of $(\theta_c-\theta_r)$ for any piston diameter when working under conditions similar to those in the gas engine just considered. Assuming that the piston thickness q would be, within limits, roughly proportional to the diameter ($d_1=2r_1$) then ($\theta_c-\theta_r$) would be approximately proportional to the diameter d_1, for any given value of the heat flow H to the piston per unit of area.

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* Air side of piston end lagged with magnesia about 2 inches thick.

If the thickness q_1 of the cylindrical wall of the piston is approximately proportional of the diameter d_1 then the sectional area of metal available for heat flow along the length of the piston ($q_1 \pi d_1$, nearly) would be nearly proportional to $d^2{}_1$. But, for any given value of H, the total heat flowing into the piston and from the piston to the jacket is proportional to $d^2{}_1$, and thus it follows that the gradient of temperature along the length of the piston would be roughly proportional to H, the heat-flow to the piston per unit of area and time. Taking the length of the piston l to be also proportional to d_1, then, for a given value of H, most likely the difference of temperature required between the piston and the cylinder liner is roughly proportional to d_1, more so since it is likely that the average thickness of the oil film between the piston and the liner would be proportional to d_1. From all these considerations it would follow that the values of θ_r and θ_c are probably not far from being proportional to the diameter of the piston for similar engines working under similar conditions. It will be recognised, of course, that the above demonstrations do *not* refer to water-cooled or oil-cooled pistons, but only to what are termed "air-cooled" pistons. This question of piston temperature is of importance because of the danger of pre-ignitions in gas and oil engines if the centre of the piston gets too hot, and also because of the liability of cracks developing in the piston due to relative expansion and excessive stresses.

In Diesel oil engines the piston is usually in close proximity to the fuel valve at the beginning of oil injection and the pulverised oil, or the flame, to a greater or less extent usually strikes the centre of the piston during injection. It is not unlikely then that the temperature of the piston and the surrounding metal has some influence on the ignition and combustion of the oil fuel. This was shown to be the case in some experiments made by the late Lieut. W. S. Burns.* Fig. 10, for example, shows rotary indicator diagrams taken by Lieut. Burns with the jacket water outlet at the temperatures given in the figure. It is easily recognised that the higher temperature facilitated the rapid combustion of the oil. The dotted horizontal lines at A, B, and C show the blast pressures used and the dotted curved lines indicate the re-expansion curves which would have been followed if no fuel had been injected. The long vertical lines represent the top dead centre and the short vertical lines near to these indicate the point of opening of the fuel valve. These experiments were made on a single-cylinder single-acting Diesel engine having a piston diameter of 12 in., stroke 18½ in. and running at 200 revolutions per minute.

It is well known that in Diesel engines the ignition and combustion of the injected oil is sometimes erratic at light loads, and in any case is nearly always later in the cycle at light loads than at heavy loads although the fuel valve opens at the same point in the cycle at all loads. This is sometimes ascribed to the cooling action of the air blast on the fuel at light loads. Whilst this cooling action undoubtedly does take place to some extent, the temperature of the centre of the piston must certainly be a contributory cause. The writer estimates that under ordinary conditions the centre of a 12 in. piston of a 4-stroke Diesel engine not specially cooled attains a temperature of about 650° F. at full load and at no load about 250° F. with continuous working.

A similar piston 20 in. diameter probably attains a temperature of 900 to 950° F. at the centre under a continuous full load. It is therefore likely that an oil which burns badly in a small Diesel engine would improve in that respect when used in a larger engine having an air-cooled piston.

With a water-cooled or an oil-cooled piston some of the tar oils, for instance, might be difficult to burn in a Diesel engine on account of the relatively cool piston, even when it burns successfully with an air-cooled piston.

In the paper referred to on p. 34, Professor Hopkinson also

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* "Some Tests on a Diesel Engine," *Trans. Engs. and Shipbds. in Scotland*, Vol. LVII, 1913–14.

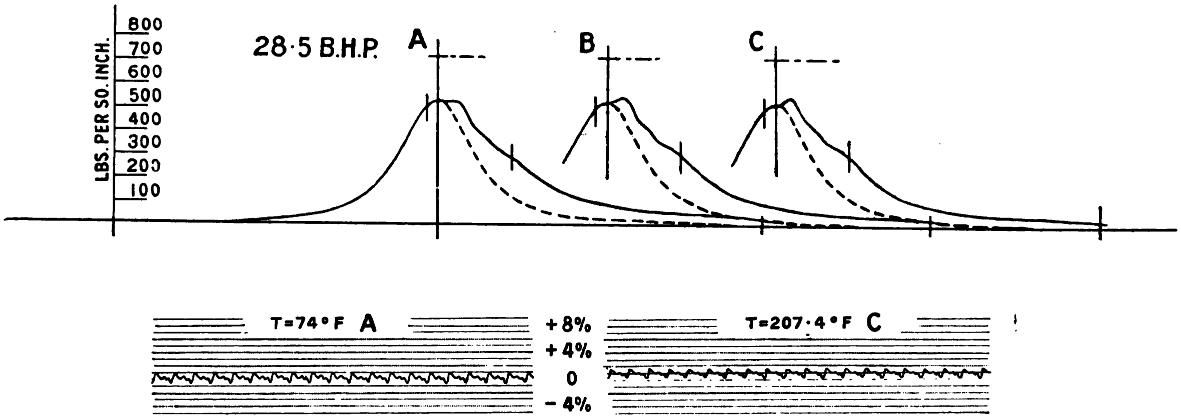

Fig. 10.—Rotary indicator diagrams and tacheograph records from a Diesel oil engine at different jacket outlet temperatures.

discusses the probable stresses induced in a flat end piston by the difference of temperature between the centre and the circumference, and concludes that these stresses are serious for a material like cast-iron.

In practice cases have occurred where the tube plate in the combustion chamber of marine boilers and locomotive boilers have shown signs of overheating. Although this tube plate is subject to severe conditions, a simple calculation shows that no overheating should take place unless the waterside of the plate has a deposit of scale or other matter or unless the circulation of water is so bad that the formation of steam bubbles prevents the water from coming into contact with portions of the plate over relatively long intervals of time.

Not only does the tube plate receive heat from the impinging gases over its flat surface but that portion of the tube which is expanded into the tube plate also receives heat, some of which has to pass into the tube plate to reach the water. Consider the conditions obtaining in a marine boiler having steel tubes 3 in. outside and 2¾ in. inside diameter, with a steel tube plate ¾ in. thick, the tubes being pitched 4 in. in horizontal and vertical rows. Taking the temperature of the gases in the combustion chamber at 2000° F., water temperature 380° F., and rate of heat transmission to plate and tube ·0015 B.Th.U. per sq. ft. per sec. per °F. difference between gas and water,

Then, area of tube plate per tube $=4\times4-\frac{\pi}{4}\times2{\cdot}75^2$

$=10$ sq. ins.

Area of tube in contact with tube plate receiving heat $=\pi\times2{\cdot}75\times{\cdot}75$

$=6{\cdot}5$ sq. in.

Total surface per tube $=10+6{\cdot}5=16{\cdot}5$ sq. in.

Heat received by tube plate* $=(2000-380)\times{\cdot}0015\times\frac{16{\cdot}5}{144}$

$={\cdot}278$ B.Th.U. per sec.

Then, Heat-flow per sec. per sq. ft. in contact with water $=\frac{{\cdot}278\times144}{10}$

$=4$ B.Th.U.

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* This neglects any transmission of heat along the length of the tube itself.

With clean surfaces in contact with boiling water the temperature of the water side of the surface at the above rate of heat transmission would be, according to Bryant's experiments, p. 212, about 13° F. above that of the water.

Also, taking the conductivity of steel at ·01 B.Th.U. per sq. ft. per sec. per °F. per ft. thick,

Then, Fall of temperature through plate* $= \dfrac{4 \times \cdot 75}{\cdot 01 \times 12}$

$= 25°$ F., roughly.

Thus, Temperature of water side of tube plate $= 380 + 13$

$= 393°$ F.

and, Temperature of gas side of tube plate $= 393 + 25$

$= 418°$ F.

Therefore, it would seem that with clean plates kept continually wet the tube plate is hardly likely to be more than about 40° F. above the temperature of the boiling water. Even if the transmission of heat were double that stated above, as might be the case in a locomotive firebox, there would be no serious rise of temperature with a clean plate kept wet, especially so if the combustion chamber or firebox happened to be of copper with its higher conductivity.

Lagging of Steam Pipes.—Steam pipes are usually covered or lagged with materials of low conductivity so as to prevent excessive losses of heat from the hot surface of the pipe. The loss of heat from a hot surface exposed to the atmosphere is due to radiation, to conduction through the air, and to the convective action of the incidental currents of air moving across the surface, and instead of a bare hot pipe the lagging introduces a different surface of larger diameter and area but having a lower temperature. It is fairly well established, however, that a surface, whether a bare hot pipe or the surface of lagging, loses heat externally to the atmosphere principally by convection.

The most economical thickness of a lagging will depend upon a multitude of circumstances, such as the cost of the material, the diameter of the pipe, the conductivity of the lagging material and the rate at which the surface loses heat to the surroundings (the "emissivity")* in comparison with the emissivity of the bare pipe. If the conditions of temperature, conductivity, costs, etc., were stated, it would be possible to calculate the most economical thickness of material.

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* Supposing all the heat to flow at right angles to the surface of the plate,

Let H = loss of heat per unit area of pipe surface per unit of [time.
r_1 = radius of pipe.
R = radius of cover or lagging.
θ_1 = temperature of pipe.
θ = temperature of surface of lagging.
θ_a = atmospheric temperature.
E = emissivity of surface of lagging.
k = co-efficient of conductivity of lagging.

Then, from eqn. 3, p. 24,

$$\theta-\theta_1=-\frac{Hr_1}{k}\log_e\frac{R}{r_1} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

also,

$$H=E\,(\theta-\theta_a)\,\frac{R}{r_1}$$

or,

$$\theta=\theta_a+\frac{Hr_1}{ER} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (2)$$

substituting in 1 from 2.

Then,

$$H=\frac{\theta_1-\theta_a}{r_1\left\{\frac{1}{ER}+\frac{1}{k}\log_e\frac{R}{r_1}\right\}} \quad . \; . \; . \; . \; . \; . \; . \; . \quad (3)$$

The value of H would be a minimum when $\frac{dH}{dR}=0$, which would give the most economical radius R, but only on the assumption that the cost, depreciation, etc., of all materials for lagging amounted to the same value. Most probably the cost C per unit length of pipe could be represented by

$$C=a+b\,(R^2-r_1^2)$$

where a and b would be constants for any given material. Given all the necessary data a few trial calculations would show the most economical thickness of the lagging. It would probably be found, however, that in the neighbourhood of the most economical thickness, the thickness may be varied between fairly wide limits without appreciably affecting the overall economy.

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* The term "emissivity," as used here, includes convection and conduction as well as radiation.

Very little information is available regarding the separate values of the conductivity and emissivity of the various materials used for lagging, and the cost, depreciation, etc., depend not only upon the materials but also to some extent upon the market conditions of supply and demand. However, it should be noted that the larger the diameter of the pipe and the higher its temperature above that of the atmosphere the greater should be the thickness of the lagging.

The usual method of experimenting with lagging material consists in supplying saturated steam to a pipe covered with a definite thickness of lagging and weighing the amount of steam condensed. Provided an efficient separator is used and none of the water of condensation is allowed to evaporate on leaving the pipe, the total heat given up by the steam at the measured temperatures can be ascertained from steam tables. For drainage purposes it is advisable to have the experimental pipe set slightly on the slope. In practice the steam passes through a steam pipe with a considerable velocity, whereas, under the above experimental conditions, the steam-flow is very small. The flow of heat through the pipe, however, is generally so small that the surface in contact with the saturated steam may be assumed to attain practically the temperature of the steam whether the velocity of the steam is large or small.

With superheated steam the experimental conditions are not quite so simple. In the first place the steam should not be allowed to condense in the pipe, and therefore it would be necessary to condense the whole of the steam in a separate condenser after passing through the experimental pipe, and to measure the inlet and outlet temperatures at the pipe in order to estimate the loss of heat from the steam. Experiment shows that, although the measured temperature of the steam at the outlet from the pipe may show many degrees of superheat, some condensation may be going on at the surface of the pipe, particularly so if the pipe is bare. The more simple method with saturated steam generally suffices unless it is desired to test whether the lagging material will stand the high temperatures associated with highly superheated steam.

Some experimenters have preferred to heat the closed space inside the experimental pipe by means of electrical resistance coils, because of the uncertainty regarding the dryness of the steam supply in the steam condensation method. The product of the drop of voltage and the amperes gives a measure of the heat lost by the pipe, and inserted thermometers measure the temperature of the enclosure. Thus,

Volts × amperes × ·057 = B.Th.U. lost per minute from the enclosure when the temperature is steady.

Experiments have been made by C. L. Norton * on a 4-in. steam pipe, 18 in. long, closed at one end by a plate welded in and the other end closed by a tightly fitting cover. This pipe was filled with cylinder oil and heated by a resistance coil through which a suitable current was sent. Special stirrers were introduced to stir up the oil briskly. The watts lost by the coil gave a measure of the loss of heat from the pipe when steady conditions had been attained. A thermometer was inserted in the oil to record the temperature. The following Table 1 gives some of the results obtained when the inside temperature was about 387° F. and the atmospheric temperature 72° F. The loss from the bare pipe was 3·8 times greater than with the worst covering and 6·3 times greater than with the best covering.

TABLE 1

Specimen No.	Name of material.	B.Th.U. per sq. ft. pipe, per sec. per deg. Fah. pipe to atmosphere.	Ratio of loss to loss from bare pipe.	Thickness of covering inches.	Weight of covering per ft. length of pipe 4 in. dia. lbs.
1	Nonpareil Cork . .	·000116	0·159	1·00	1·7
2	,, ,, Octagonal	·000121	0·172	0·80	1·0
3	Magnesia. . . .	·000130	0·177	1·12	2·2
4	Imperial Asbestos .	·000132	0·180	1·12	2·8
5	W.B.	·000139	0·189	1·12	3·7
6	Asbestos Air Cell . .	·000147	0·200	1·12	2·2
7	Manville Infusorial Earth	·000148	0·202	1·50	—
8	Magnabestos . . .	·000154	0·210	1·12	3·0
9	Moulded Sectional .	·000159	0·217	1·12	3·6
10	Asbestos Fire Board .	·000176	0·241	1·12	2·2
11	Calcite	·000191	0·261	1·12	4·1
12	Bare Pipe . . .	·000733	1·000	—	—

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* "The Protection of Steam-Heated Surfaces," *Trans. Amer. Soc. Mech. Engs.*, Vol. 19, 1898.

No. 1. Granulated cork pressed in a mould at high temperature, and then submitted to a fireproofing process.

No. 2. Similar in composition, but made up of several strips of cork instead of two semi-cylindrical sections.

No. 3. Moulded sectional cover composed of about ninety per cent carbonate of magnesia.

No. 4. Sheets of asbestos paper indented so as to keep the sheets from coming in contact with one another thus enclosing air in the spaces.

No. 5. Wool felt on a lining of asbestos paper.

No. 6. Thin sheets of asbestos paper fluted and stuck together with silicate of soda.

No. 7. Plastic covering of infusorial earth.

No. 8. Moulded cover containing about 45 per cent of carbonate of magnesia and a considerable percentage of carbonate of lime.

No. 9. Mainly sulphate of lime and about 20 per cent carbonate of magnesia, with thick sheet of felt board on outer surface.

No. 10. Similar to *No.* 6 except that cells were larger and contained more silicate of soda.

No. 11. Sectional moulded cover composed mainly of sulphate of calcium. It had an outer layer of felt board.

Tests were made with the pipe bare under the different conditions given below, the inside temperature being about 387° F.

TABLE 2

Condition of Bare Pipe.	B.Th.U per sq. ft. pipe per sec. per deg. Fah. pipe to atmosphere.
New pipe	·000634
Fair condition	·000734
Rusty and black	·000752
Cleaned with caustic potash inside and out	·000734
Painted dull white	·000757
Painted glossy white	·000637
Cleaned with potash again	·000734
Coated with cylinder oil.	·000736
Painted dull black	·000762
Painted glossy black	·000641

These results would seem to indicate that the greater portion of the loss of heat from a hot surface to the atmosphere is due to convection of the air, and that only a small proportion is due to radiation and conduction.

Some tests were made with the bare pipe in various positions in the room to see whether the position had much influence on the loss of heat. The largest variations were not more than about 10% from the results with the apparatus in the standard position in the centre of the room.

Experiments have been made on lagging materials by W. N. Bolam and T. Grieve * in the engineering laboratory at Edinburgh University. The piping used was a piece of ordinary 3-in. wrought-iron steam piping, 8 ft. 10⅝ in. long, with a mean external diameter of 3·53 in., giving a total area of uncovered pipe surface under test of 8·13 square feet. Special precautions were taken to ensure dry saturated steam at the inlet, and the water of condensation was weighed after passing through a cooling coil. Special tests were made to allow for the condensation due to end connections. The following Table 3 represents some of the results obtained :—

TABLE 3

† Type of Lagging.	B.Th.U. per sq. ft. pipe per sec. per deg. Fah. steam to atmosphere.					Weight of covering per sq. ft of uncovered pipe surface lbs.	Thickness of covering inches.
	At steam gauge pressures, lbs. sq. in.				Mean value.		
	25	50	70	100			
Uncovered pipe	·00061	·00082	·00089	·00089	·0008	—	—
Slag wool	·000145	·000161	·000172	·000183	·000165	3·07	1·25
Hair and felt	·000136	·000164	·000161	·000161	·000156	2·40	1·20
Felt	·000195	·000211	·00022	·00022	·000211	2·33	0·90
Magnesia	·000139	·000156	·000164	·000167	·000157	2·68	1·50

These results indicate that the condensation in the uncovered pipe was from 4 to 5½ times greater than in a covered pipe.

With a small electrically heated tester C. R. Darling has made some experiments ‡ with magnesia, with a plastic

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* *Engineering*, Aug. 7th, 1903, p. 171.

† All the coverings except the slag wool were of the detachable type.

‡ *Engineering*, July 9th, 1909.

covering, and with different densities of slag wool. The magnesia was composed of 85% magnesium carbonate and 15% asbestos fibre. The plastic covering had some vegetable fibres in its composition. The values given below in Table 4 have been selected as representative of the character of the results obtained with different internal temperatures and different thicknesses of lagging.

TABLE 4

Material.	Internal temperature, °F.	Atmospheric temperature, °F.	B.Th.U. per sq. ft. pipe per sec. per deg. Fah. pipe to atmosphere.			
			Thickness in inches.			
			1	1½	2	2½
Magnesia . .	250 300 400 500	70 70 70 70	·00014 ·00014 ·00013 ·00012	·000120 ·000110 ·000098 ·000091	·000100 ·000097 ·000084 ·000073	·000087 ·000081 ·000069 ·000061
Plastic covering .	250 350 450	70 70 70	·00017 ·00015 ·00016	·000130 ·000125 ·000125	·000100 ·000096 ·000100	·000083 ·000081 ·000084

Tests were made with slag wool of definite thickness but of different densities. The results indicated a minimum loss of heat in the neighbourhood of density 30 lb. per cubic foot.

In some further experiments* Mr. Darling embedded thermojunctions in the lagging so as to give the temperatures at various depths in the material. In one case the lagging was covered with untreated asbestos cloth and afterwards the surface was painted with aluminium paint. The reduction in radiation due to the smooth-painted surface caused a reduction of the loss of heat by about 7%, even though the rise of temperature of the surface was about 25° F. because of the smaller heat-flow and smaller gradient of temperature through the lagging when painted. The internal temperature was the same under both conditions at about 370° F.

A large number of tests on various kinds of lagging for steam pipes have been conducted at Manchester University by

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* *Engineering*, Sept. 22nd, 1911.

Mr. Bradley* with particular reference to 1-in. steam pipes. Some of the tests were carried out on a steam-condensing apparatus up to 200 lb. per sq. in. pressure and a check was obtained in some cases by means of a pipe heated internally by an electrical-resistance coil. Altogether 57 samples were tested and some of the results obtained are represented in Fig. 11. Here the heat lost per sq. ft. of pipe surface per hour is shown on a base of temperature difference between

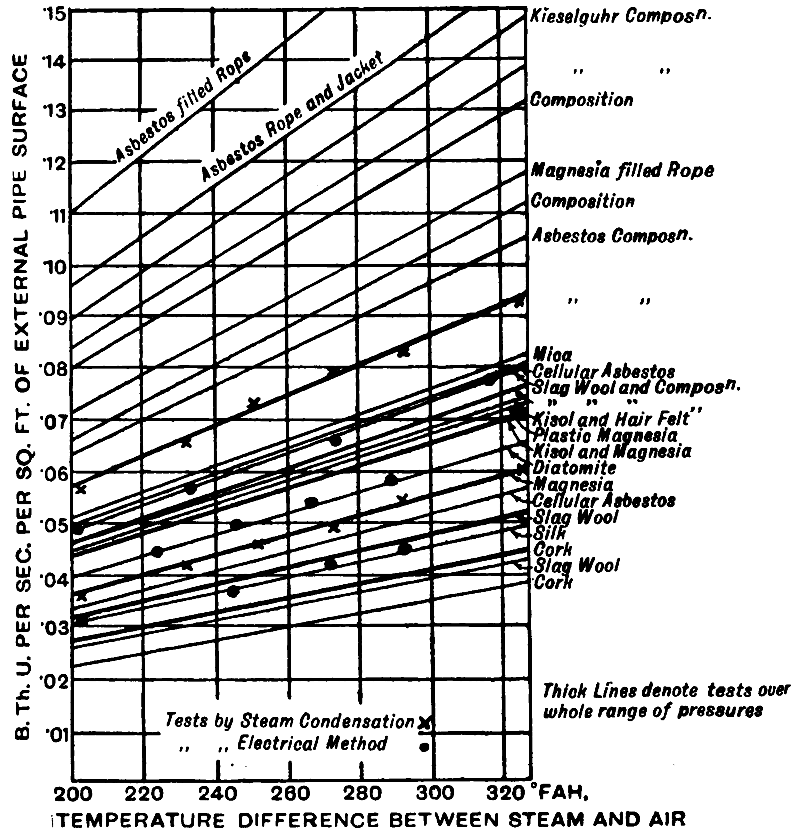

Fig. 11.—Loss of heat from a 1-in. covered steam pipe.

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* "Humidity and Ventilation in Weaving Sheds." Second and Final Report of Committee. *Home Office Report* (Cd. 5566), 1911.

the steam and the air. So as not to complicate the diagram only a few of the experimental points are shown. It is seen, however, that within the range of the experiments there is a straight line relation between the loss of heat and the temperature.

From the results of these experiments the report says that plastic or semi-plastic coverings are less satisfactory as heat insulators than other forms and, generally speaking, the lightest material is the best insulator.

Professor Petavel* and Mr. C. N. Lander have made some experiments on the transmission of heat from bare and from lagged pipes, using a steam-condensation apparatus. Some of the results obtained are given in Table 5.

TABLE 5

External dia. of pipe inches.	† Loss by Bare Pipe { B.Th.U. per sq. ft. pipe per sec. per deg. Fah. steam to atmosphere.			
	Steam at atmos. press. (Temp. diff. 152° F.).	Steam at 100 lbs. sq. in. (Temp. diff. 278° F.).	Steam at 200 lbs. sq. in. (Temp. diff. 328° F.).	Superheated steam at 500° F. (Temp. diff. 440° F.).
0·2	·00110	·00150	·00160	·00167
0·5	·00093	·00119	·00125	·00136
1·0	·00081	·00103	·00109	·00120
2·0	·00072	·00094	·00099	·00109
3·0	·00070	·00092	·00097	·00106
6·0	·00066	·00088	·00093	·00103
12·0	·00065	·00087	·00092	·00102

It will be noted that the rate of loss of heat from the bare pipes decreases as the diameter increases, but it remains practically constant for diameters above 6 in. It was estimated that with the 1-in. uncovered pipe losing heat at the rate ·00083 with steam at 212° F., 15% of this total loss was due to radiation, 4% to the conduction of the air, and the remaining 81% to convection of the surrounding air. With a bare pipe having a good machined surface the loss by radiation was reduced to 7% of the total loss and with a pipe polished it was reduced to between 3 and 4%. Therefore a polished surface has only a comparatively small influence on the total loss of heat.

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* *Brit. Ass. Report*, 1912. *Trans. Manchester Ass. Engs.*, 1915.

† These values refer to an oxidised iron surface. For a perfectly polished surface about ·000055 at 212° F. and about ·000083 at 500° F. should be subtracted.

It hardly seems likely that the amount of heat radiated from a bare pipe per unit of area is much dependent upon the diameter, and therefore the changes in the loss of heat with the diameter, shown in the preceding Table 5, were most likely due to a decrease in the convective action of the air with increasing diameter.

With any loosely packed lagging the conductivity depends to a large extent upon the density. The above experimenters found that slag wool, for instance, showed a minimum loss of heat with a density of 10 to 12 lb. per cubic ft., corresponding to a density of about $\frac{1}{14}$ that of the individual fibres. The loss of heat under these conditions was about $\frac{1}{7}$ of that with the bare pipe.

A lengthy series of tests on pipe coverings have been made by L. B. McMillan,* who used a 5-in. steel pipe, 16 ft. long, closed at the ends and filled with gas-engine cylinder oil. The oil was heated by a current sent through resistance coils fixed in the pipe, and the oil was kept moving by a special screw stirrer driven by a small motor. The temperatures of the oil and the outside surface of the pipe were measured by means of thermo-junctions. About 5 in. at each end of the pipe was covered with 85% magnesia covering 1-in. thick, leaving exactly 15 ft. length of the pipe for testing, and to allow for the heat lost by these end portions a similar short pipe was made and separately tested. The heat lost from the 15 ft. length was therefore the difference between that lost by the long pipe and that lost by the short one, and this was obtained from the watts expended when the temperatures had attained the steady condition.

A series of tests were made, both with the pipe bare and covered, to find the fall of temperature between saturated steam and the outside surface of the pipe. With the uncovered pipe this was not more than 3° to 4° F. when the steam temperature was about 350° F., and when covered was less than 1° F., and so from this it could be concluded that the

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* *Journal Amer. Soc. Mech. Engs.*, Jan., 1916.

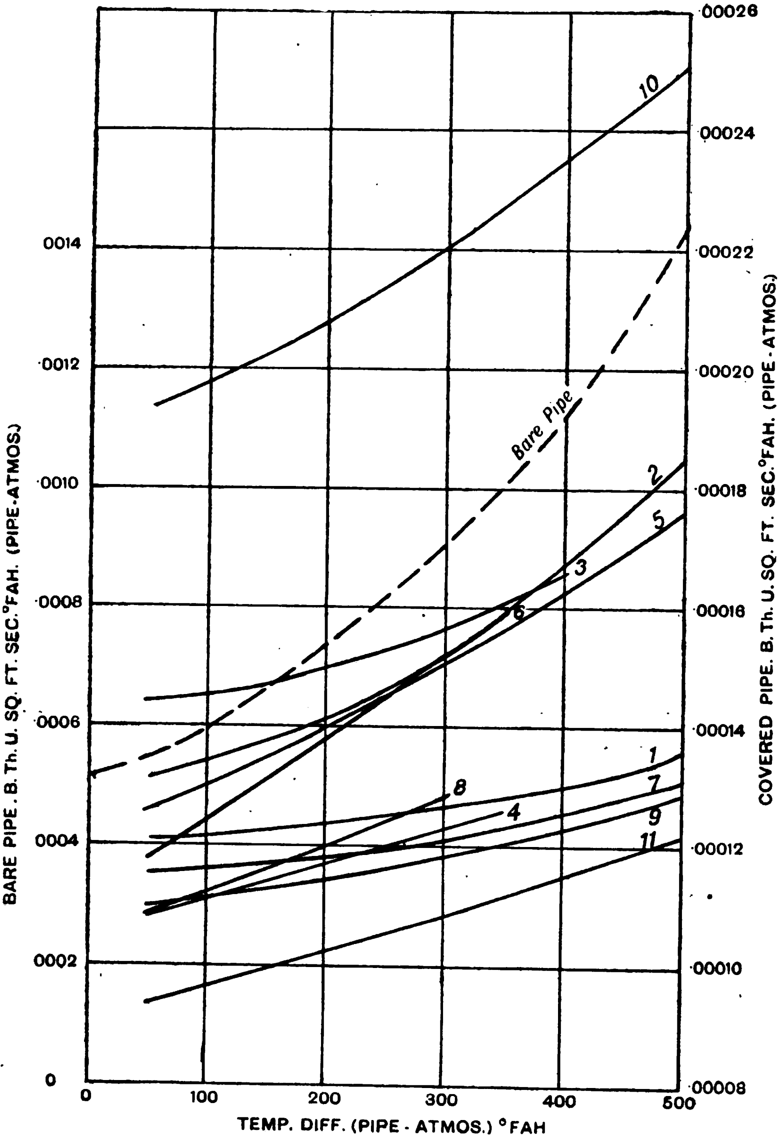

Fig. 12.—Loss of heat from a covered and a bare 5-in. pipe.

pipe temperature was practically the same as that of saturated steam when flowing through the pipe.

Tests were made with eighteen different coverings and the results of some of these have been shown in diagram form in Figs. 12 and 13. In Fig. 12 the left-hand scale refers to the dotted curve for the bare pipe and the right-hand scale to the pipe when covered with the various coverings enumerated below. Fig. 13 shows the rate of heat loss with variations of the thickness of 85% magnesia covering, the full lines representing the results with plastic covering when the temperature difference between the pipe and the atmosphere was 500° F., 300° F., and 100° F., and the dotted lines refer to similar conditions but with sectional covering.

The following are the coverings to which the numbers on the curves in Fig. 12 refer. These coverings were all purchased in the open market, and the statements as to whether the covering was recommended for high or low-pressure steam or super-heated steam were furnished by the manufacturers.

(1) J. M. 85% Magnesia. Moulded sectional covering for high-pressure steam pipes, 85% by weight of magnesium carbonate and remainder principally asbestos fibre. Weight per lineal foot 2·92 lb. and thickness 1·08 in.

(2) J. M. Indented. Layers of asbestos felt with indentations, about 1¼ in. diameter and ⅛ in. deep, spaced very close to each other in staggered rows. For use on high-pressure steam pipes. Weight per foot 3·46 lb. and thickness 1·12 in.

(3) J. M. Moulded Asbestos. For low and medium steam pressures. Asbestos fibre and other fireproof material. Weight per foot 5·53 lb. and thickness 1·25 in.

(4) J. M. Wool Felt. Sectional covering made of layers of wool felt with an interlining of two layers of asbestos paper, for low-pressure steam and hot-water pipes. Weight per foot 2·59 lb. and thickness 1·1 in.

(5) Carey Serrated. Successive layers of heavy asbestos felt having closely spaced indentations, for high-pressure steam pipes. Weight per foot 5·66 lb. and thickness 1 in.

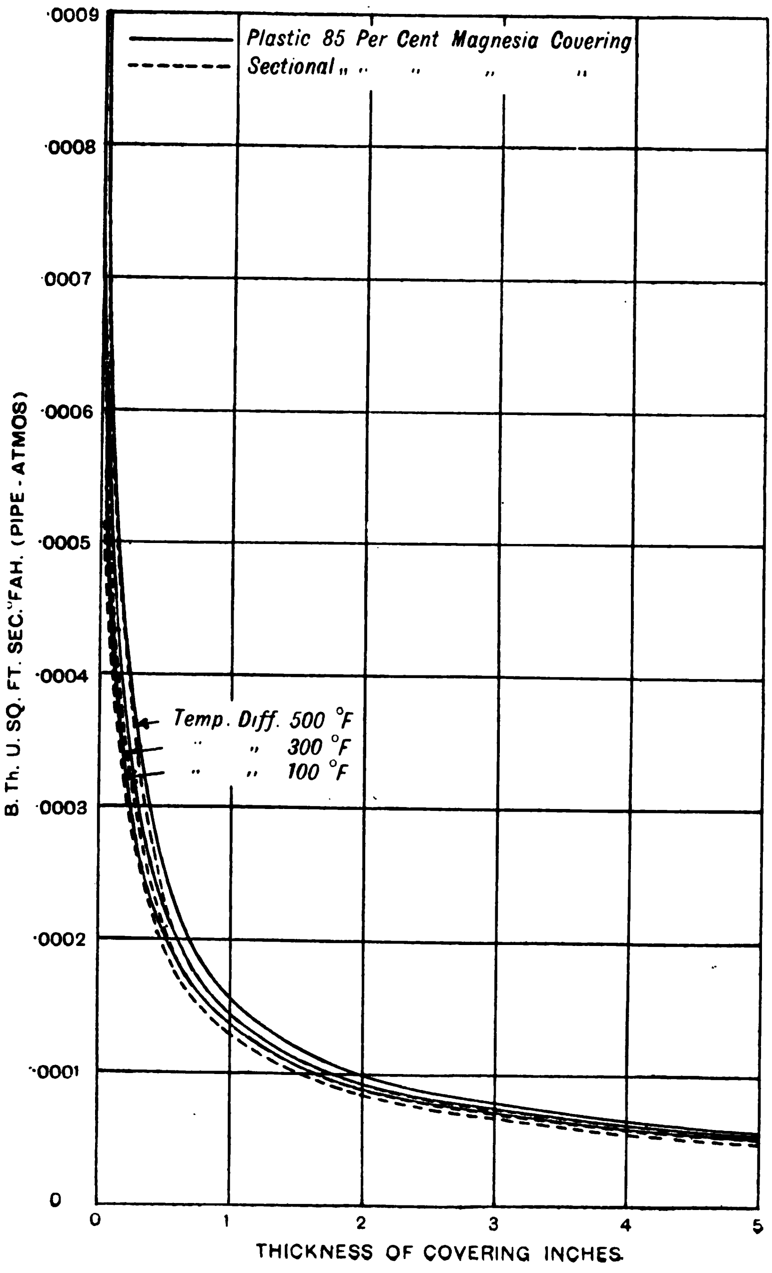

FIG. 13.—Variation of loss of heat with thickness of 85% magnesia covering on 5-in. pipe.

(6) Carey Duplex. Alternate layers of plain wool felt and corrugated asbestos paper firmly bound together. Corrugations run lengthwise of pipe and air cells approximately $\frac{1}{4}$ in. deep. For low-pressure steam and hot-water pipes. Weight per foot 1·79 lb. and thickness ·96 in.

(7) Carey 85% Magnesia. Similar in composition to (1). For high-pressure steam pipes. Weight per foot 2·74 lb. and thickness 1·1 in.

(8) Sall-Mo. Wool Felt. Similar to (4) except without interlining of asbestos paper. For low-pressure steam and hot-water pipes. Weight per foot 3·73 lb. and thickness 1·01 in.

(9) Nonpareil High Pressure. Moulded sectional covering consisting mainly of silica in the form of diatomaceous earth, for high-pressure steam and superheated steam pipes. Weight per foot 2·96 lb. and thickness 1·16 in.

(10) J. M. Asbestos Fire-Felt. Asbestos fibre loosely felted together, forming a large number of air spaces. For high-pressure steam and superheated steam pipes. Weight per foot 3·75 lb. and thickness ·99 in.

(11) J. M. Asbestos Sponge Felted. Made from a thin felt of asbestos fibre and finely ground sponge forming a very cellular fabric. Forty-one of these sheets per inch of thickness. Air spaces are formed between the sheets in addition to those in the felt itself. For high-pressure steam and superheated steam pipes. Weight per foot 4·04 lb. and thickness 1·16 in.

It will be seen from all the results of these various experiments on lagging materials that magnesia is a good heat insulator and that asbestos is much inferior in this respect. Both are fire-resisting and durable when properly handled.

Magnesia is sometimes mixed with carbonate of lime or with sulphate of lime (plaster of Paris) to obtain a cheaper covering, which is often sold under the name of magnesia, but both carbonate and sulphate of lime are inferior to magnesia as heat insulators.

Cork material may be durable when the steam-temperature is not high, but is not fireproof.

Felt is liable to char even at moderate temperatures, and in case of fire may be dangerous, but otherwise it is a good insulator.

In general it may be said that lagging which is kept moist, by leakage of steam for instance, is much inferior to dry lagging.

Some representative values of the coefficient of conductivity for various insulating materials as given in Table 24, pp. 227–8.

Very little experimental data is available respecting the loss of heat from a lagged pipe carrying superheated steam. It may be stated, however, that the rate of loss is much the same as with saturated steam at the same temperature, for the reason that the temperature of the metal of the pipe in either case is only one or two degrees below the temperature of the flowing steam, and it is the temperature of the pipe, other things the same, which determines the amount of the loss.

With respect to the flow of superheated steam it is sometimes important to be able to make a preliminary estimate of the fall of temperature in a length of pipe under stated conditions.

Let T_1=steam temperature at pipe inlet.
T_2= ,, ,, ,, ,, outlet.
t_a=atmospheric temperature.
d=outside diameter of pipe.
l=length of pipe.
h=heat transmission per unit of pipe outside surface, per unit of time, per degree difference between the steam and air.
W=weight of steam-flow per unit of time.
s=specific heat of superheated steam.
$=\cdot 5$, say.

In a length of pipe δx the fall of temperature is δT, and if steady conditions* have been attained,

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* When superheated steam is made to flow through a covered pipe initially cold a long period usually elapses before a steady state of temperature along the length of the pipe is attained.

Loss of heat $= Ws\,\delta T = h\,(T - t_a)\,\pi d\,\delta x$, nearly.

or,
$$\frac{\delta T}{T - t_a} = \frac{h\pi d}{Ws}\,\delta x.$$

Thus, integrating,

$$\log_e (T_1 - t_a) - \log_e (T_2 - t_a) = \frac{h\pi d}{Ws}\,l \quad . \quad . \quad . \quad . \quad . \quad (4)$$

For example, if the pipe diameter is 6 in. outside, covered with magnesia 1½ in. thick, h would be about ·0001 B.Th.U. per sq. ft. per sec. per deg. Fah. difference of temperature, the pipe being in an ordinary room at 60° F. (If exposed to the wind and weather h might easily be double the above value.) $T_1 = 600°$ F., $W = 3{\cdot}5$ lb. per sec., $s = {\cdot}5$, $l = 100$ ft.

$$\text{Then, } \log_e (600 - 60) - \log_e (T_2 - 60) = \frac{{\cdot}0001 \times \pi \times {\cdot}5}{3{\cdot}5 \times {\cdot}5} \times 100$$
$$= {\cdot}00897$$
$$\text{or, } 6{\cdot}2916 - {\cdot}00897 = \log_e (T_2 - 60)$$
$$T_2 - 60 = 535$$
$$T_2 = 595°\text{ F.}$$

It will be seen from equation 4 that the temperature T_2 would be lower the smaller the value of W. To a first approximation calculation shows that the drop of temperature would be nearly inversely proportional to W. Thus, with $W = 1$ lb. per sec., $T_2 = 583°$ F., and with $W = {\cdot}5$ lb. per sec., $T_2 = 567°$ F. In the above calculation the pipe flanges are assumed to be covered as effectively as the plain pipe, but usually the flanges are fitted with detachable coverings or are left bare. Some additional allowance would be necessary according to the condition of the flange coverings and their number.

CHAPTER III

CONVECTION AND RESISTANCE TO THE FLOW OF FLUIDS

THE convection of heat refers to the carrying of heat from one place to another by the motion of fluids. A simple case of convection is obtained when water is heated in a vessel from below ; the hot water, having a lower density than the colder water, tends to rise and currents are thereby induced which cause the distribution of the hotter particles of water among the colder and thus tends to equalize the temperature throughout the vessel. A more complicated case of convection occurs when heat is being transmitted to a tube surface from a fluid moving through the tube at high velocity. Whirling or eddying motions in the flowing fluid cause a rapid distribution of the particles or molecules among one another, and in practically all cases where heat has to be transmitted from or to a fluid, this convection is depended upon to facilitate the transference of heat. But the flow of a fluid through a uniform tube can only be obtained when there is a difference of pressure or head between the two ends sufficient to overcome the resistance to the flow, and thus there is likely to be some relation between the resistance to the flow and the transference of heat. For this reason the laws governing the resistance offered to the flow of a fluid will first be discussed.

The general laws of the motions of fluids in parallel channels was first clearly enunciated by Professor Osborne Reynolds* and proved by him for water by direct experiment. He used two methods in his experiments, (1) visual observations on the flow through glass tubes, and (2) measuring the resistances and velocities through pipes of different diameters and with different temperatures of water. In the experiments by visual observation glass tubes of 1 in., ½ in., and ¼ in. diameter were specially selected to be as uniform in bore as possible. They were all about 4 ft. 6 in. long and had trumpet-shaped entrances so that the water might enter without disturbance. The tubes were immersed in a large glass tank as represented in Fig. 14, and the water was allowed to flow through the tubes from the tank, the rate of flow being controlled by the outlet cock A. A constant level was maintained in the tank during an experiment. Arrangements were also made so that a streak of coloured water from the vessel B entered the centre of the tube along with the clear water.

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* *Trans. Roy. Soc.*, 1883 ; also *Scientific Papers*, Vol. 2.

The general results of the experiments were as follows: When the velocities of flow were low the streak of coloured water remained in a straight line through the tube, as in Fig. 14, but if the water in the tank had not quite settled to rest before entering the tube, the streak would shift about the tube at sufficiently low velocities, but there was no appearance of the breaking up of the streak. The flow of fluids in this manner is sometimes called "viscous flow" or "stream line motion."

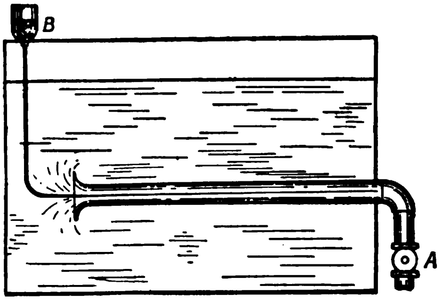

FIG. 14.—Apparatus showing character of water flow through tubes or pipes.

As the velocity of the water was gradually increased the colour band or streak would all at once mix up with the surrounding water near the outlet end of the tube and any increase of the velocity beyond this critical value caused the point of breakdown to approach the entrance end, but with no velocities that were tried did it reach this end. By viewing the tube by the light of an electric spark it was found that the breakdown of steady motion was associated with the formation of whirling or eddying motion, somewhat as represented in Figs. 15 (a) and (b).

The sudden change from stream line to eddying flow in fluids is also exhibited by the smoke coming from the lighted end of a cigar or the bowl of a tobacco pipe when in still air. For the first few inches the smoke rises in a stream, but this suddenly breaks down and the flow then becomes confused or eddying. The character of eddying flow in a mild form is also exhibited in the wreaths of smoke leaving an ordinary chimney.

With water at a constant temperature and as still as could be obtained in the tank, it was found that the critical velocities at which the eddies showed themselves were almost exactly in the inverse ratio of the tube diameters, and within the range of the experiments from 5° C. to 22° C. it was found that the critical velocity diminished as the temperature increased.

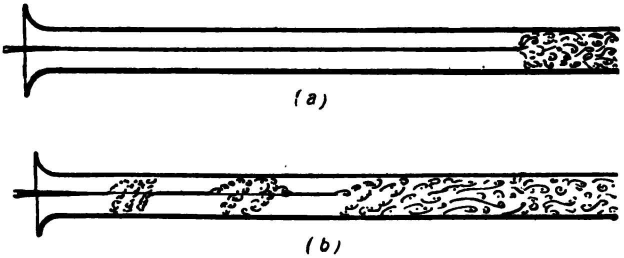

FIG. 15.—Breakdown of stream-line flow into eddying flow.

If v_c=higher critical velocity,* feet per second.

d=diameter of tube, feet.

T=temperature of water, deg. Cent.

$$\frac{\mu}{\rho}=\frac{\dagger\text{viscosity}}{\text{density}}=\sigma=\left(\frac{1}{1+\cdot03368\text{T}+\cdot000221\text{T}^2}\right)^{\ddagger} \quad . \quad . \quad . \quad . \quad (1)$$

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[*] In all cases the term "velocity" refers to the mean axial velocity through the pipe unless otherwise stated.

[†] Refer to p. 63 for the definition and discussion of viscosity. $\frac{\mu}{\rho}$ is sometimes called the kinematical viscosity of the fluid.

[‡] From Poiseuille's experiments, taking σ at 0° C.=1.

It was found that, to these units and for the values of σ given below,

$$v_c = \cdot 246\frac{\sigma}{d} \text{ ft. per sec. } \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (2)$$

This gives the " higher critical velocity " as distinct from the " lower critical velocity," u_c, discussed subsequently. The values in Table 6 give the higher critical velocities for water at 0° C. for different diameters of pipe.

TABLE 6

Diameter of pipe, inches . .	0·5	1·0	1·5	2·0
v_c at 32° F., ft. per sec. . .	5·9	2·95	1·97	1·47

The following relative values of $\sigma = \frac{\mu}{\rho}$ indicate how the critical velocity varies with the temperature :

$T = 32°$ F. $= 0°$ C. $\sigma = 1$
$T = 62°$ F. $= 16·5°$ C. $\sigma = \cdot 616$
$T = 100°$ F. $= 37·75°$ C. $\sigma = \cdot 385$
$T = 212°$ F. $= 100°$ C. $\sigma = \cdot 152$.

For example, the above values show that at 100° F. the higher critical velocity for water is about ·385 of that at 32° F. If it were desired to express μ and ρ in absolute values, then the values μ given on p. 65 could be used, and the equations 1 and 2 could be altered accordingly.

Great care was necessary in the above experiments in preserving steady and uniform conditions in the tank so that consistent results could be obtained, as the critical velocity was found to be unstable under disturbed conditions. This region of instability was found to lie between the higher critical velocity v_c and the lower value u_c, at which latter value any previously existing eddies would gradually die out as the flow continued through the tube. The values of u_c, however, could not be determined by the visual methods because the eddying motion when once started broke up the streak of coloured water and it was then impossible to see when the eddies died out. It was possible, however, to deduce the value of u_c by the second method, i.e. by measuring the resistance to the flow between two points in a tube, and this method was also used by Professor Reynolds to determine the general relations between resistance and velocity of flow.

For this purpose lead pipes $\frac{1}{4}$ in. and $\frac{1}{2}$ in. internal diameter and about 16 ft. long were used. Only the last 5 ft. of the pipes were used for measuring the resistance, and it was intended that the preceding 10 ft. or so of pipe would give a chance for any induced eddying motion to die out before the water reached the 5 ft. length. The water was taken from the ordinary town's main and care was taken to preserve constant conditions during each experiment. The average velocity of flow was calculated by measuring the quantity of water in a measuring vessel. The resistance to the flow was measured as shown in Fig. 30, p. 96, and a special cathetometer was used to measure the difference of level very accurately. On plotting a curve connecting the loss of head due to the resistance with the mean velocity it was found that for any given tube the points lay on a straight line up to a certain velocity, and that this line passed through the zero of co-ordinates. Above this velocity the points fell on or near to a smooth curve, suggesting that the loss of head was proportional to v^n, i.e. $y=av^n$, where a and n are constants for the particular conditions. If this law is true, then,

$$\log y=\log a+n \log v,$$

which is the equation to a straight line, and plotting the logs of y and v Professor Reynolds showed that the above expression represented a general law for a fluid like water.

It is generally found that for water the results obtained are as represented in Fig. 16. The plotted points lie on a straight line up to some point A for gradually increasing velocities starting from zero, the gradient of this line being unity. That is, up to the point A, $y=bv$.

This point A is taken to be the " higher " critical velocity v_c, or the point at which initially steady motion breaks down into eddying motion. Beyond this velocity the resistance is somewhat erratic and increases at a very rapid rate until some point such as B is reached. This rapid increase of resistance between A and B is probably due to the birth of eddying motions and part of the head y has to be spent in creating the kinetic energy involved in these eddying motions. The velocity at B is about 1·3 times that at A and beyond this a definite straight line BC is obtained having a gradient n, and giving the law $y=av^n$. When the velocities are decreased the points return along CB and continue to the point D of intersection with the line OA. This indicates that the eddies in the water die out at the velocity D. The velocity at D is considered to be the " lower " or " true " critical velocity, $u_c = \cdot 0387\frac{\sigma}{d}$.

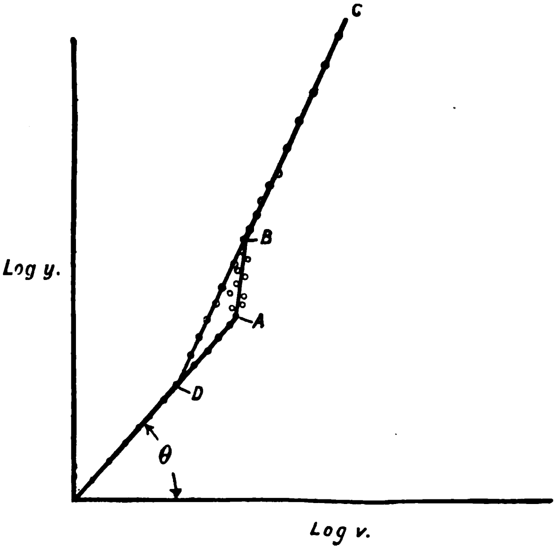

FIG. 16.—Law connecting resistance y with velocity of flow v.

The values in Table 7 give the lower critical velocities.

TABLE 7

Diameter of pipe, inches	0·5	1·0	1·5	2·0
u_c at 32° F., ft. per sec.	·928	·463	·31	·232

Professor Reynolds deduced from his experiments the following values for n above the critical velocities.

	n
Lead pipe	1·79
Varnished pipe	1·82
Glass pipe	1·79
New cast-iron pipe ..	1·88
Incrusted cast-iron pipe	2·00
Cleaned cast-iron pipe ..	1·91

Below the critical velocity $n=1$ in all cases.

Viscosity.—The resistance offered to the flow of a fluid generally depends to some extent upon the viscosity of the fluid. The viscosity μ may be defined as follows : Suppose two planes in a fluid to be at a distance d apart and one plane moves parallel to the other with a relative velocity* v, a tangential force F is exerted on the planes per unit area given by—

$$F=\mu\frac{v}{d} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (3)$$

where μ is the viscosity of the fluid.

In this definition it is assumed that there is no slip of the fluid over the planes considered, that is, if the planes are solid surfaces the fluid in contact with these planes is supposed to move with them at the same velocity. Both theory and experiment indicate the probability of this assumption being justified, except in extreme cases of gases at very low pressures, and in some cases with liquids.

When relative motion takes place in a gas as explained in connection with equation 3, the kinetic theory of gases indicates that the resistance to motion or the viscosity of a gas is due to the diffusion of molecules from layer or layer, that is, some molecules jump from the more rapidly moving layers into the slower-moving layers and lose momentum, and similarly, a corresponding number jump in the reverse direction and gain momentum. It is these changes of momentum which is supposed to constitute the resistance which a gas offers to relative motion.

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* Although natural diffusion of the molecules occurs, no eddying or convective flow is supposed to take place between the two planes.

A reference to Meyer's *Kinetic Theory of Gases* will also show how it has been proved that, above very low pressures, the value of μ for a given gas is practically independent of the pressure, and that it varies approximately as some power a of the absolute temperature τ of the gas, i.e., as τ^a.

It has been found that the product of the viscosity μ and the specific heat of the gas at constant volume S_v is proportional to the conductivity of the gas. The values given in Table 8 have been taken from Professor Dalby's* paper on "Heat Transmission" which in turn were obtained from data given in Landolt and Börnstein's tables. The values for μ_0 in lb., feet, seconds units have been added by the author.

TABLE 8

	Values of a in τ^a	Viscosity at 0° C.		Specific heat at 0° C.		$\frac{S_p}{S_v}$	$\mu_0 S_v \times 10^6$ C.G.S. units.	Conductivity at 0° C. C.G.S. units $k \times 10^6$	$\frac{k}{\mu_0 S_v}$
		C.G.S. units $\mu_0 \times 10^5$	Lbs., ft., sec. units $\mu_0 \times 10^7$	At constant pressure S_p	At constant volume S_v				
	1	2	3	4	5	6	7	8	9
Air .	·76	17	3·54	0·24	0·170	1·41	29	55	1·90
CO_2 .	·92	14	2·90	0·20	0·154	1·30	21	31	1·48
N_2 .	·74	16	3·34	0·24	0·170	1·41	27	52	1·93
O_2 .	·79	18	3·75	0·21	0·149	1·41	27	56	2·08
CO .	·74	16	3·34	0·24	0·170	1·41	27	50	1·85
H_2 .	·63	8	1·67	3·40	2·410	1·41	193	330	1·71

The values of μ_0 given in column 2 are in dynes per square centimetre and express the frictional force F when v is 1 cm. per sec. and d is 1 cm. Column 3 gives the results in pounds per square foot when the velocity is 1 ft. per sec. and the distance d is 1 ft.

The kinetic theory of gases also indicates that the conductivity k of a gas is proportional to the product μS_v. Columns 8 and 7 give the values of k and $\mu_0 S_v$ respectively, while column 9 gives the ratio $\frac{k}{\mu_0 S_v}$. In general it is found that this ratio decreases as the number of atoms in the molecules increases, and comparing the value for CO_2 with the other values in column 9 indicates the relation.

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* *Proc. Inst. Mech. Engs.*, Oct.–Dec., 1909, p. 936.

According to the experiments made by Gibson and Grindley* on the resistance to the flow of air through tubes of small bore the viscosity of air is independent of the pressure and increases with the temperature T, being expressed by—

For air, $\mu=(355{\cdot}3+1{\cdot}168T-{\cdot}00248T^2)10^{-9}$ ft., lb., sec., °C. units or, approximately between 0° C. and 100° C.

$$\mu=(358+{\cdot}932T)10^{-9} \text{ ft., lb., sec., °C. units.}$$

The molecules of a liquid are subject to cohesive forces and are much less free to roam in space than are the molecules of a gas. It is probable, then, that cohesion in a liquid plays a part in producing viscosity. It is found that in liquids the viscosity decreases with an increase of temperature, which is the reverse of what occurs in a gas. In liquids, therefore, the smaller cohesion at higher temperatures more than counterbalances the increased diffusity. The viscosity of water μ given by Poiseuille's experiments † is, when T is in deg. Fahr.,

$$\mu=\frac{{\cdot}00003716}{{\cdot}4712+{\cdot}01435T+{\cdot}0000682T^2}, \text{ ft., lb., sec., °F. units,}$$

or, when T is in deg. Cent.,

$$\mu=\frac{{\cdot}00003716}{1+{\cdot}03368T+{\cdot}000221T^2}, \text{ ft., lb., sec., °C. units.}$$

Table 9 gives some values of μ for water, and indicates how rapidly μ decreases with increase of temperature.

TABLE 9

Temperature.		μ ft., lb., sec., units.
°F.	°C.	
32	0	·00003716
50	10	·00002735
68	20	·00002109
86	30	·00001685
104	40	·00001376
122	50	·00001150
140	60	·00000971

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* Gibson's *Hydraulics and its Applications.*

† *ibid.*, p. 12.

Experiments by Hauser indicate that above 32° F. the viscosity μ for water increases slightly with increase of pressure.

Lack of space prevents any detailed consideration of the apparatus and methods in use for the experimental determination of the viscosity of a fluid. For liquids there are several methods available which are based on the rate of flow through capillary tubes. For an explanation of these reference might be made to the following article in *The Petroleum World*, June and July, 1913, or The National Physical Laboratory *Collected Researches*, Vol. XI, 1914. *On the Methods and Apparatus used in Petroleum Testing*, Part II, " Viscometry," by W. F. Higgins.

For the methods available in the case of gases reference could be made to *The Kinetic Theory of Gases*, by Meyer.

Resistance to the Flow of Fluids through Tubes or Channels of Uniform Cross-section.—To obtain a full and comprehensive idea of the laws relating to the resistance offered to the flow of fluids it is necessary to discuss the subject at some length. Taking it that the fall of pressure δP, or resistance R_o, per unit length of pipe when a steady state of motion occurs depends only upon the diameter of the pipe d, upon the viscosity μ and density ρ of the fluid, and on the velocity v of flow, Professor Reynolds deduced a general expression in the following manner: Assuming that each of the above factors raised to some power only enter into the general expression, then,

$$R_o \propto d^a \rho^b \mu^c v^n.$$

Each of these factors may be expressed in the dimensions of mass M, length L, and time T, and the product should be equal to the dimensions of a force per unit of length.

Thus, $$R_o \propto \left[L\right]^a \left[\frac{M}{L^3}\right]^b \left[\frac{M}{LT}\right]^c \left[\frac{L}{T}\right]^n \propto \left[\frac{ML}{LT^2}\right]. \quad . \quad . \quad . \quad (4)$$

Equating the indices of similar dimensions—

$$\therefore a-3b-c+n=o$$
$$b+c=1$$
$$c+n=2$$

Thus, $a=n-1,\ b=n-1,\ c=2-n$

$$\therefore \mathrm{R}_o \propto d^{n-1}\rho^{n-1}\mu^{2-n}v^n,$$

or the resistance R per unit of surface is given by—

$$\mathrm{R}=\frac{\mathrm{R}_o}{\pi d}\propto d^{n-2}\rho^{n-1}\mu^{2-n}v^n,$$

or, again, the fall of pressure $\delta\mathrm{P}$ in length l is given by

$$\delta\mathrm{P}=\frac{\mathrm{R}\pi dl}{\frac{\pi d^2}{4}}=\frac{4\mathrm{R}l}{d}$$

$$=qd^{n-3}\rho^{n-1}\mu^{2-n}v^nl=gy\rho \quad \ldots\ldots (5)$$

where q is a constant, g the acceleration due to gravity, and y the height of a column of fluid equivalent to $\delta\mathrm{P}$.

If $\frac{y}{l}$ is the loss of head per unit length of pipe and $\sigma=\frac{\mu}{\rho}$, sometimes called "kinematical viscosity,"

then, $$\frac{y}{l}=\frac{q}{g}\sigma^{2-n}\frac{v^n}{d^{3-n}}=\frac{\mathrm{B}^n}{\mathrm{A}}\sigma^{2-n}\frac{v^n}{d^{3-n}} \quad \ldots\ldots (6)$$

where A and B are constants introduced to allow for slight variations of q with different values of n.

If the units are feet, seconds and deg. Cent.

Then, for water, $\mathrm{A}=1{\cdot}93\times10^6$, and $\mathrm{B}=36{\cdot}9$.

The above expression for $\frac{y}{l}$ may be modified thus,

$$\frac{y}{l}=\frac{\mathrm{B}^n}{\mathrm{A}}\sigma^{2-n}\left(\frac{w}{a\rho}\right)^n\frac{1}{d^{3-n}}$$

where w = weight flowing per unit of time.

a = area of flow $=\frac{\pi}{4}d^2$ for circular pipe.

$v\rho a=w$.

Therefore, if the weight w and diameter d were kept constant the hydraulic gradient $\frac{y}{l}$ would vary with $\sigma^{2-n}\left(\frac{1}{\rho}\right)^n$.

On referring to the values of σ for water given on p. 65 it would be seen that σ decreases rapidly with increase of temperature, and since ρ decreases slightly above 4° C., the factor $\left(\frac{1}{\rho}\right)^n$ increases with temperature. Taking n for a smooth pipe at about 1·8, for example, then,

For water at

$$62^\circ \text{ F.}=16\tfrac{1}{2}^\circ \text{ C.}, \left\{\sigma^{2-n}\left(\frac{1}{\rho}\right)^n\right\}=\cdot907\times\cdot000587=\cdot000533.$$

For water at

$$100^\circ \text{ F.}=37\tfrac{3}{4}^\circ \text{ C.} \quad ,, \quad =\cdot826\times\cdot000594=\cdot00049.$$

For water at

$$212^\circ \text{ F.}=100^\circ \text{ C.} \quad ,, \quad =\cdot686\times\cdot000632=\cdot000434.$$

The above values indicate that, with a smooth pipe, the resistance to the flow of a given weight of water through a pipe of given diameter decreases by about 8 per cent when the temperature is increased from 62° F. to 100° F. Experiments confirm this result. For example, Mr. Mair,* experimenting on a smooth brass tube 1½ in. diameter and 25 ft. long, found a decrease of resistance of about 8% between the same temperatures, the index n being 1·795.

If, however, the pipe is very rough, then usually $n=2$ above the critical velocity, and $\sigma^{2-n}=1$, under which conditions it would follow that the resistance would vary as $\left(\frac{1}{\rho}\right)^2$ and would therefore increase slightly with the temperature of water above 4° C.

In applying equation 5 to the flow of compressible fluids such as air it may be modified in the following manner:

Assuming a perfect gas, where $\frac{\text{PV}}{\tau}=\text{C}$, then,

$$\rho=\frac{1}{\text{V}}=\frac{\text{P}}{\text{C}\tau},$$

and

$$\delta\text{P}=\frac{f_1\mu^{2-n}v^n l}{d^{3-n}}\left(\frac{\text{P}}{\text{C}\tau}\right)^{n-1} \quad \dots\dots\dots \quad (7)$$

If $\frac{w}{\rho a}=\frac{w}{a}\frac{\text{C}\tau}{\text{P}}$ be inserted for v, then equation 7 becomes

$$\frac{\delta\text{P}\times\text{P}}{\tau}=f_1\mu^{2-n}\frac{l}{d^{3-n}}\text{C}\left(\frac{w}{a}\right)^n \quad \dots\dots\dots \quad (8)$$

It will be seen that making $n=2$ reduces this expression to the approximate form given on p. 85.

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* *Proc. Inst. Civil Engs.*, Vol. LXXXIV, 1885–86, Part II.

According to Professor Gibson* the following values of n for air apply above the critical velocities.

	n
Lead pipe ·125 in. diam., very smooth inner surface, Gibson	1·28
„ „ ·25 „ „ probably moderately rough, Brix.	1·49
Cast-iron pipe 7·87 in. diam., Stockalper	1·775
„ „ 11·75 „ „ Brix.	1·77
Wrought-iron pipe 3·25 in. diam., Brix.	1·825
„ „ 5·2 „ „ Brix.	1·81
„ „ 5·9 „ „ Stockalper	1·81
„ „ 6·75 „ „ Brix.	1·79

Professor Gibson further suggests the following values of n as suitable for the flow of air above the critical velocities through cast-iron and wrought-iron pipes.

Diameter of pipe, inches.	3	5	7	9	12
n	1·83	1·81	1·79	1·775	1·77

Below the critical velocity $n=1$.

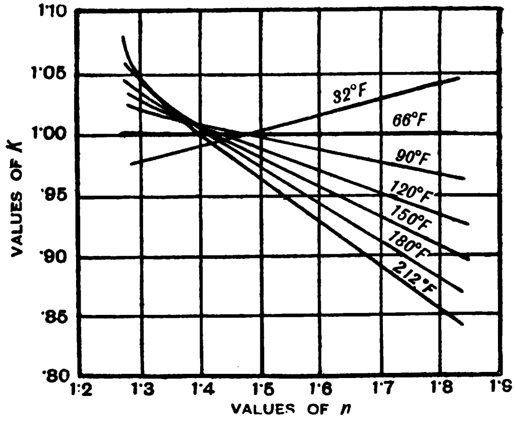

FIG. 17.—Relation between K_1 and n in equation 9.

Equation 5 has been modified by Professor Gibson to the following for the flow of air :

$$\delta p = \frac{\cdot 00000125 K_1 p_m^{n-1} v_m^n l}{6\cdot 6^n d^{3-n}} \text{ lbs. per sq. in.} \quad . \quad . \quad . \quad (9)$$

---

* *Phil. Mag.*, 1909, p. 398.

where K_1 has the values given in the graph in Fig. 17.

p_m = mean pressure in pipe, lbs. per sq. in.
v_m = mean velocity in pipe, ft. per sec.
l = length of pipe, ft.
d = dia. of pipe, ft.

It is obvious that $\frac{w}{a} \times \frac{1}{\rho} = \frac{w}{a} \times \frac{C\tau_m}{p_m}$ may be substituted in place of v_m, where w is lb. per second, a is area of flow, sq. ft., $C = \cdot 37$ if p_m is in lb. per sq. in., and τ_m is the mean absolute temperature in deg. Fahr.

For ordinary purposes it may be taken that $\mu \propto \tau^{\frac{3}{4}}$ for a gas, where τ is the absolute temperature. Considering the probable variation of δP in equation 8, p. 68, with the absolute temperature τ, all other values constant, then,

$$\begin{aligned} \delta P &= f_1' \frac{l}{d^{3-n}} \left(\frac{w}{a}\right)^n \frac{\tau}{P} \tau^{\frac{3}{4}(2-n)} \\ &= f_1' \frac{l}{d^{3-n}} \left(\frac{w}{a}\right)^n \frac{1}{P} \tau^{(2 \cdot 5 - \cdot 75n)} \quad \ldots\ldots \quad (10) \end{aligned}$$

Therefore for a gas,

$$\left.\begin{array}{l} \text{If } n=1 \text{ for “stream line” flow, } \delta P \propto \tau^{1 \cdot 75} \\ \text{If } n=1 \cdot 8 \text{ for eddying flow, } \delta P \propto \tau^{1 \cdot 15} \\ \text{If } n=2 \text{ for eddying flow, } \delta P \propto \tau \end{array}\right\} \text{All other values constant.}$$

Reverting again to the general equation 5, p. 67, for the motion of fluids deduced by Professor Osborne Reynolds, the equation may also be modified as follows :—

Let R = resistance to the flow per unit surface of pipe.

Then, $R\pi d\,\delta x = \delta P \frac{\pi}{4} d^2$ for a circular pipe,

$$\begin{aligned} \text{or, } R &= \frac{\delta P d}{4\,\delta x} \\ &= \frac{q}{4} d^{n-2} \left(\frac{\rho}{\mu}\right)^{n-2} v^{n-2} \rho v^2 \\ &= \frac{q}{4} \rho v^2 \left(\frac{vd}{\frac{\mu}{\rho}}\right)^{n-2} \quad \ldots\ldots\ldots\ldots \quad (11) \end{aligned}$$

or, again, $\delta P = \frac{4R\,\delta x}{d} = q\frac{\rho v^2}{d}\left(\frac{vd}{\frac{\mu}{\rho}}\right)^{n-2}\delta x$

or, since $\rho v = \frac{w}{a} = \frac{\text{weight of flow per unit of time}}{\text{area of section of flow}}$

$$\text{then, } \delta P = q\left(\frac{vd}{\frac{\mu}{\rho}}\right)^{n-2}\frac{\delta x}{d}\left(\frac{w}{a}\right)^2\frac{1}{\rho} \quad \ldots\ldots\ldots \quad (12)$$

This equation was verified between certain limits by Professor Reynolds for the flow of water through tubes.

If this law is generally true, then for any two dissimilar fluids (1) and (2), the pressure difference for a given length of a given pipe will, if the index n be identical for the two fluids, be related by the equation,

$$\frac{\delta P_1}{\delta P_2} = \frac{\rho_1}{\rho_2}\left(\frac{\sigma_1}{\sigma_2}\right)^{2-n}\left(\frac{v_1}{v_2}\right)^n$$

$$\text{or, } \frac{y_1}{y_2} = \left(\frac{\sigma_1}{\sigma_2}\right)^{2-n}\left(\frac{v_1}{v_2}\right)^n \quad \ldots\ldots\ldots \quad (13)$$

using σ to represent $\frac{\mu}{\rho}$, and y the loss of head ; and again, for a given pipe, at identical velocities,

$$\frac{y_1}{y_2} = \left(\frac{\sigma_1}{\sigma_2}\right)^{2-n} \quad \ldots\ldots\ldots\ldots \quad (14)$$

Professor A. H. Gibson* has shown that the above laws hold for the case of brine solutions of various densities. His experiments were made on the flow of brine through a series of similar cold-drawn copper pipes, whose mean diameters were ·751, ·998 and 1·5 in., and a galvanised iron pipe 2·009 in. diameter.

Four series of experiments were carried out on each tube. In the first series (a) plain water was used ; in the remainder, (b), (c), and (d), brine solutions of various strengths, in series (b) 4%, (c) 11%, and (d) 18% by weight of salt in 100 parts of solution.

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* "The Resistance to the Flow of Brine Solutions through Pipes," *Proc. Inst. Mech. Engs.*, Jan.–Feb., 1914, p. 201.

The measured values of μ, ρ, and σ at 15° C. relative to water were as follows :—

Sodium Chloride in solution.	ρ	μ Relative to Water.	σ
Water	1·000	1·000	1·000
4 per cent . . .	1·030	1·051	1·021
11 „ . . .	1·082	1·203	1·110
18 „ . . .	1·134	1·455	1·281

The experimental results are shown graphically in Fig. 18, in which the logarithms of loss of head in feet of water are plotted as ordinates on a base showing logarithms of the mean velocity of flow, feet per second. The tangents of the inclinations give the values of n. For any one of the pipes the lines for all the solutions are sensibly parallel, showing that n is practically independent of the viscosity. In the smooth copper pipes n appears to increase slightly with the diameter as follows :—

Diameter of pipe, inches . .	·751	·998	1·500
Value of n	1·785	1·790	1·795

The mean value is 1·79.

For the galvanised pipe $n=1{\cdot}965$.

If the equations 13 and 14, p. 71, are true it follows that the loss of head with a given solution at a given mean velocity should be greater than for water at the same velocity in the ratio $\left(\frac{\sigma_b}{\sigma_w}\right)^{\cdot 21}$ for the 1-in. pipe ($n=1{\cdot}79$).

Thus with the 4% solution $\left(\frac{\sigma_b}{\sigma_w}\right)^{\cdot 21}=1{\cdot}004$

11 „ „ $=1{\cdot}022$

18 „ „ $=1{\cdot}053$

The dotted straight lines in Fig. 18 have been drawn in accordance with the above values, and it was found that they represented the experimental results at least as well as any other lines which could be drawn.

In the 2-in. pipe the index of $\left(\frac{\sigma_b}{\sigma_w}\right)$ is $(2-1{\cdot}965)={\cdot}035$,

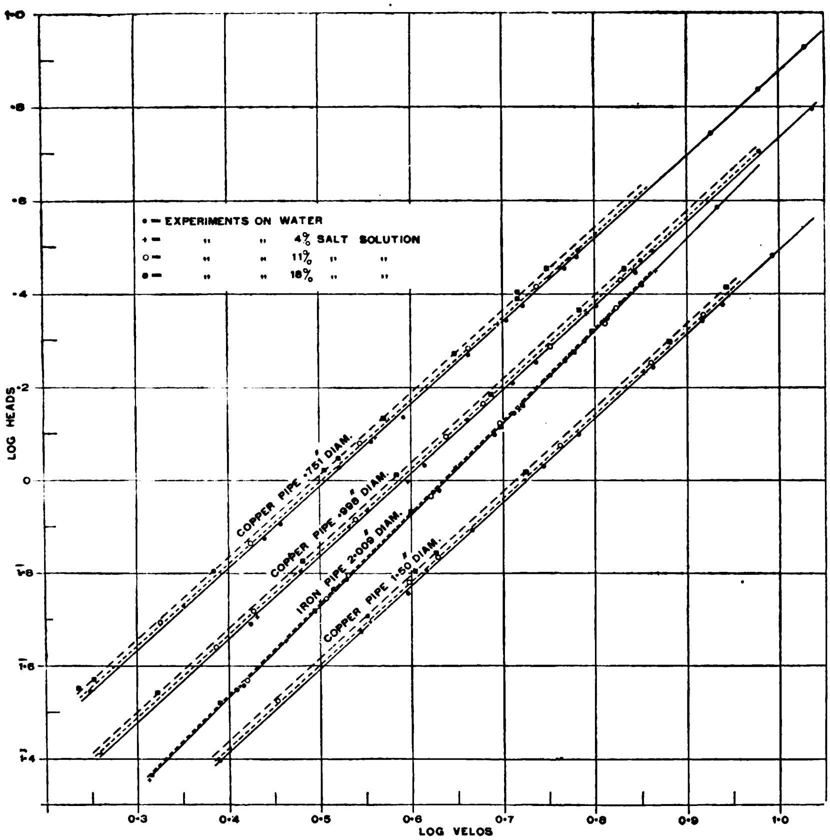

FIG. 18.—Relations between resistance and velocity of flow for brine solutions of various densities.

which is very small. It follows that the resistance should only be very slightly influenced by the value of σ. Thus with the 18% solution the loss of head should only be 1·009 times as great as with water, a conclusion which was confirmed by the results of the experiments on this pipe. In the above-mentioned paper a series of tables are given containing values of μ, both for NaCl and for CaCl solutions of various temperatures and specific gravities.

It was pointed out on p. 68 that, if $n=2$, then the resistance should be independent of the value of σ. In practice, under ordinary conditions of operation, a certain amount of corrosion occurs in some cases, or a deposit occurs in other cases at the pipe surface, roughening this surface, and then the resistance usually becomes nearly proportional to the square of the velocity.

Lord Rayleigh has shown, however, that the solution given by equation 11, p. 70, is only a particular case of a general law of resistance of bodies immersed in fluids moving relatively to them, under the assumption that this resistance depends only on the linear dimensions of the body and on the velocity, density, and the ratio of viscosity to density of the fluid. This relation may be expressed as

$$R=\rho v^2\phi\left(\frac{vd}{\sigma}\right) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

where R is the resistance per unit area and $\phi\left(\frac{vd}{\sigma}\right)$ represents a function of the one variable $\left(\frac{vd}{\sigma}\right)$.

Stated in words this signifies that the resistance R per unit area of "rubbing"* surface is proportional to ρv^2 for any two or more fluids when $\left(\frac{vd}{\sigma}\right)$ is the same for each fluid, provided the surfaces relatively to which the fluids move are geometrically similar, which similarity, of course, necessarily must extend to those irregularities in the surfaces which constitute roughness. Stated generally, it may be said that fluids have similar motions when the value of $\left(\frac{vd}{\sigma}\right)$ is the same for each, that is, if for any fluid v_1, d_1, and σ_1 signify respectively the velocity, diameter of pipe, and the ratio of the viscosity to the density of the fluid, and v_2, d_2, and σ_2, the respective values for another fluid, these fluids would flow with similar motions when,

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* In using the word "rubbing" it is *not* intended to mean that the fluid necessarily slips over the surface.

$$\frac{v_1 d_1}{\sigma_1} = \frac{v_2 d_2}{\sigma_2}.$$

Dr. T. E. Stanton* and Mr. J. R. Pannell have verified this law under very variable conditions for the cases of pipes through which air or water is allowed to flow. They also showed by experiments on pipes over a wide range of velocities that the index law of Osborne Reynolds, equation 5, p. 67, does not exactly represent the results, for as the velocity increased beyond moderate values, the value of n had to be increased slightly to enable the results to fit in with the formula.

Another interesting point to be deduced from equation 15, p. 74, and verified by Dr. Stanton's experiments for air and for water, is that for any pipe through which a fluid flows, the value of $\left(\frac{vd}{\sigma}\right)$ will be the same at the critical velocity no matter what fluid is used, provided, of course, the roughness of the pipe is similar in all cases. This would suggest that, given the critical velocity of any one fluid, that for any other fluid flowing through a similar pipe of any diameter could be calculated approximately if the value of σ were known for both fluids.

If the units for $\left(\frac{vd}{\sigma}\right)$ are the centimetre, gram., second, or the foot, pound, second, μ being measured in dynes per sq. cm., or in poundals per sq. ft. respectively, then, $\left(\frac{vd}{\sigma}\right)=2200$ approximately at the lower critical velocity, and at the higher critical velocity appears to give $\left(\frac{vd}{\sigma}\right)=3200$ approximately. If the viscosity μ is measured in pounds per sq. ft. then, $\left(\frac{vd}{\sigma}\right)$ $=2200\times 32{\cdot}2=71{,}000$, or $3200\times 32{\cdot}2=103{,}000$, at the lower and higher critical velocities respectively. It is sometimes useful to remember that the value of $\sigma=\frac{\mu}{\rho}$ for air is about 13 times that for water at ordinary atmospheric temperatures.

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* "Similarity of Motion in Relation to the Surface Friction of Fluids," *Trans. Roy. Soc.*, A, Vol. 214; or "Collected Researches," *National Physica Laboratory*, Vol. XI, 1914.

From Stanton and Pannell's results for smooth pipes Dr. C. H. Lees* has deduced by trial the following expression,

$$R=\rho v^2\left\{\cdot 0765\left(\frac{\sigma}{vd}\right)^{\cdot 35}+\cdot 0009\right\} \quad . \quad . \quad . \quad . \quad . \quad (16)$$

the units being centimetre, gram, second, with μ in dynes per sq. cm., R being in dynes per sq. cm. If the units are pound, foot, second, and μ in poundals per sq. ft., R also being in poundals per sq. ft., the same expression should hold.

Since $\delta P=\frac{4Rl}{d}$ for a circular tube.

Then, using equation 16

$$\delta P=\frac{\rho v^2 l}{d}\left\{\cdot 306\left(\frac{\sigma}{vd}\right)^{\cdot 35}+\cdot 0036\right\} \quad . \quad . \quad . \quad . \quad . \quad (17)$$

An examination of these equations shows that as the velocity and diameter increase, and as the value of $\frac{\mu}{\rho}=\sigma$ decreases, the pressure difference δP varies more nearly as $\rho\frac{v^2 l}{d}$.

It is also evident that if the resistance varies as v^2, as it nearly does in rough pipes above the critical velocity, the resistance must then be practically independent of the value of the term $\left(\frac{vd}{\sigma}\right)$.

Dr. Stanton† has also experimentally demonstrated these similar motions as regards the distribution of velocity from the centre to the surface of pipes through which air was allowed to flow, both for pipes artificially roughened and for smooth pipes. The apparatus consisted of an air fan arranged to discharge into a horizontal pipe 3·5 metres (11·5 ft.) long.

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[*] *Proc. Roy. Soc.*, A, Vol. 91.

[†] "The Mechanical Viscosity of Fluids," *Proc. Roy. Soc.*, A, Vol. 85, 1911; or "Collected Researches," *National Physical Laboratory*, Vol. VIII, 1912.

This pipe was connected by a bend to a vertical pipe 5·5 metres (18 ft.) high, and the experimental portion, 61 cm. (2 ft.) long, was at the upper extremity of this vertical pipe. It was found by experiment that this length of outlet was sufficient to set up the limiting distribution of velocity across the pipe as long as the diameter did not exceed 7·5 cm. (2·95 in.). The values of the velocity v were determined from the difference between the pressure in a small Pitot tube facing the current and that in a small orifice in the side of the pipe. The mouth of the Pitot tube was rectangular in section, the internal dimensions being: in the direction of the pipe radius ·25 mm. (·0099 in.) and in the direction of the pipe length 1·2 mm. (·047 in.). The Pitot tube was fixed to a micrometer and could be moved across the pipe. The pressure difference was measured by a sensitive oil and water tilting gauge.

The pressure difference along the selected length of pipe was measured as indicated in principle on p. 96. This was sufficient for the purpose since no variation of static pressure across the section of the pipe could be found up to a distance of 2 mm. (·079 in.) from the walls.

As mentioned previously, from equation 15, p. 74, it follows that if the frictional resistance to the flow happened to vary as v^2 with ρ practically constant, then $\frac{vd}{\sigma}$ would be constant. To simplify the problem Dr. Stanton experimented on two pipes of diameters 7·35 cm. (2·9 in.) and 5·08 cm. (2·0 in.) in which the friction varied as v^2, and consequently the friction per unit area of surface at the same velocity of flow was found to be the same for the two pipes. These pipes were obtained by cutting a screw-thread on the inside surface, having the pitch and depth of the threads arranged so that the surfaces were geometrically similar.

To test the constancy of the velocity distribution in a radial direction two sets of observations were made with the air velocities at the pipe centres 2220 cm. (72 ft.) and 546 cm. (17·9 ft.) per second respectively. The two sets of reduced velocities for the 7·35 cm. pipe are shown plotted in Fig. 19 and this illustrates the general nature of the distribution of axial velocity in a circular pipe. These velocities are well above the critical value, and although the mean motion at any point taken over a sufficient length of time is parallel to

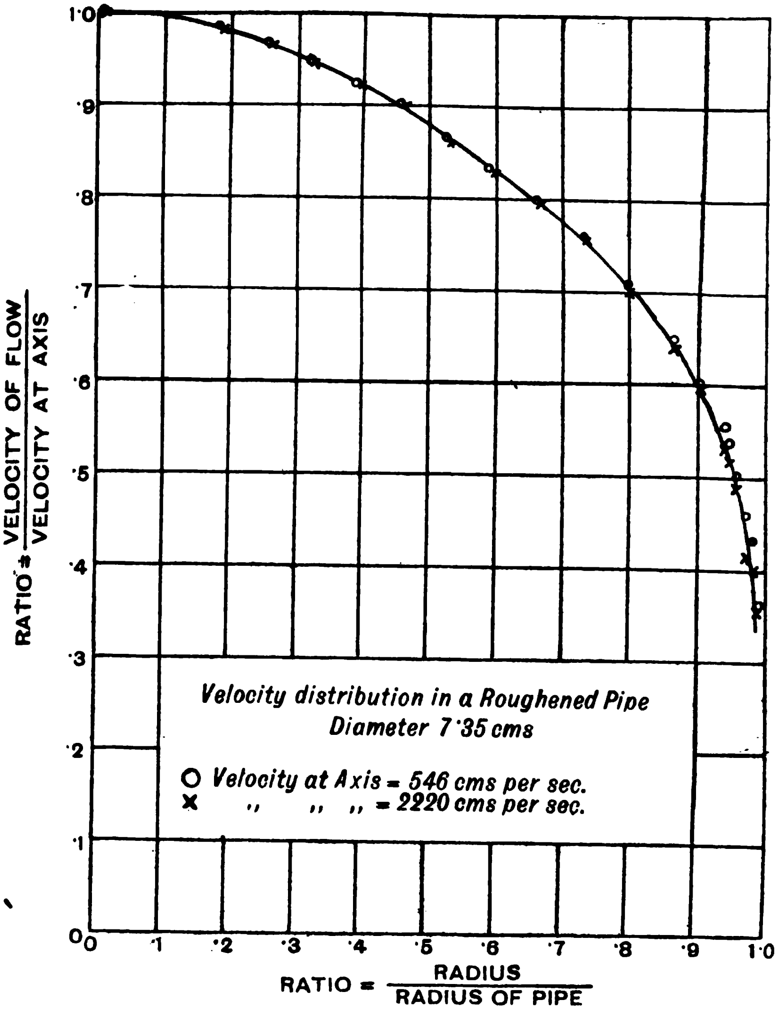

Fig. 19.

the axis of the pipe, it is really made up of a succession of motions crossing the pipe in different directions due to the eddying character of the flow.

As regards the effect of the diameter of the pipe on the velocity distribution it was found that if in the two pipes the velocities at the centres were made proportional to the corresponding diameters the curves were precisely similar. This result is shown in Fig. 20.

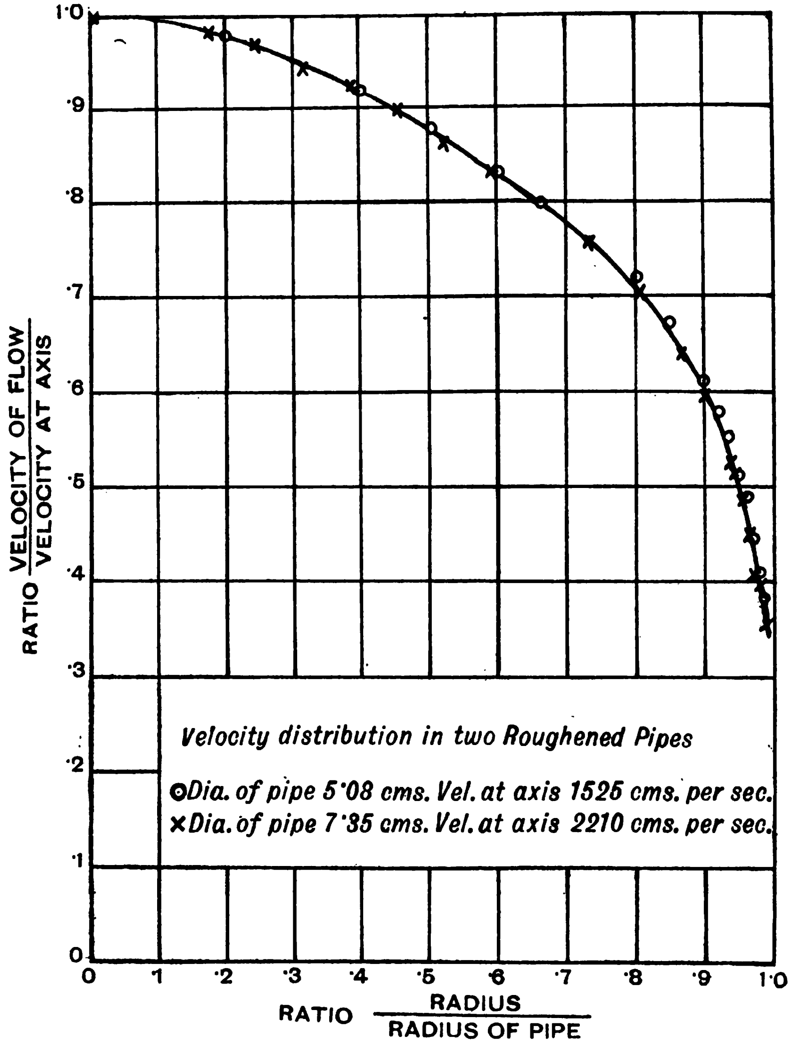

Fig. 20.

Further experiments were made on two smooth brass pipes,* of approximately the same diameters as the roughened ones.

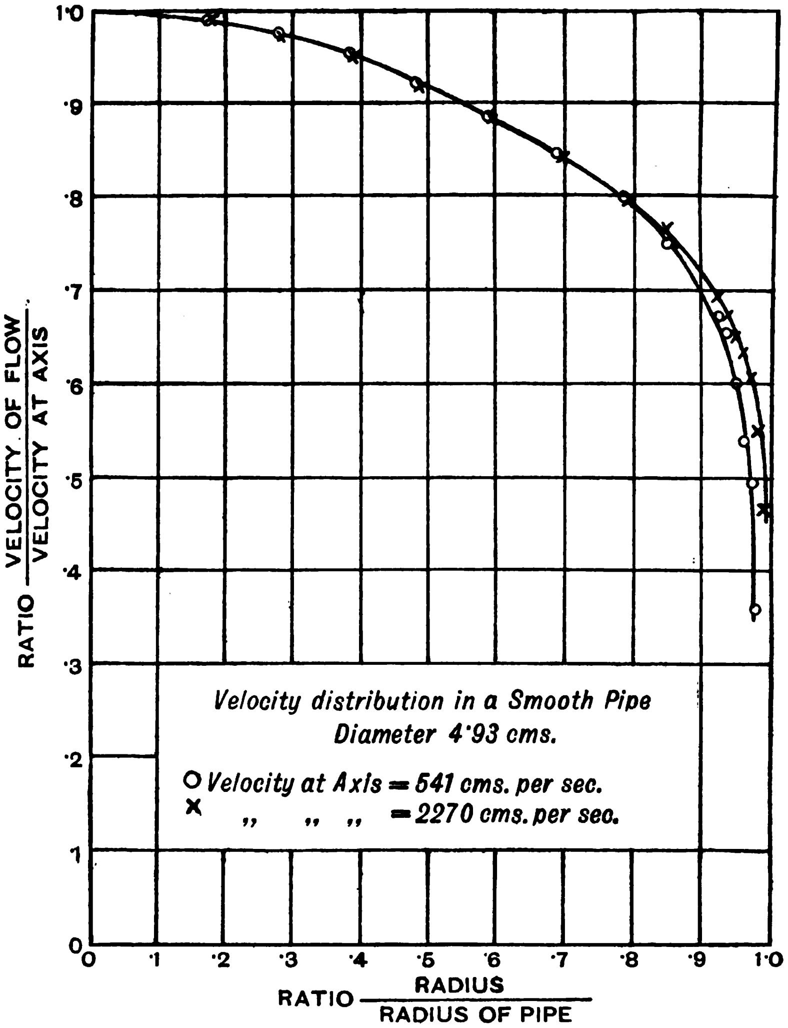

Fig. 21.

These were 4·93 cm. and 7·4 cm. (1·94 and 2·91 in.) diameter. Two sets of observations were made on the 4·93 cm. pipe with centre velocities 2270 cm. (74·5 ft.) and 541 cm. (17·8 ft.) per second, and the distribution of velocities are shown plotted in Fig. 21. It will be seen that, for a region extending from the centre up to a value of the radius of about 0·8r, the distributions are identical, but that beyond this radius the curves separate, indicating apparently a region of more or less stream line flow near the walls. Respecting the effect of the size of the pipe a precisely similar investigation to that adopted for the roughened pipes showed that, for two pipes in which the centre velocities were proportional to the respective diameters, the velocity distributions were strictly similar throughout the central portion of the flow up to a radius of about 0·8r, but beyond this radius they were different.

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* In smooth pipes the resistance would not be proportional to v^2 but to some lower power of v as indicated by the values of n on p. 69.

But even with smooth pipes it would be expected from equation 15, p. 74, that similarity of motions over the whole section would obtain when $\frac{vd}{\sigma}$ has the same values for the two pipes, that is, if σ is constant, when v is inversely proportional to d. To illustrate this similarity, experiments were made on the distribution of velocity in the two smooth pipes in which the centre velocities were 1525 cm. (50 ft.) per second in the 4·93 cm. pipe and 1017 cm. (33·4 ft.) per second in the 7·4 cm. pipe, using air at practically the same pressure and temperature. The results obtained are plotted in Fig. 22, showing that identical curves of velocity were obtained from the centre to the walls.

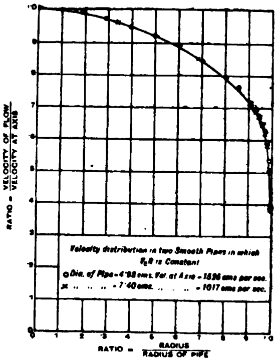

Fig. 22.—Velocity distribution in two smooth pipes where *vd* is constant.

It is therefore seen that the results of experiments so far available indicate that the general equations relating to the flow of fluids are expressions of fundamental laws.

In boiler and condenser problems relating to the resistance to the flow of fluids it is often difficult to state precisely the working conditions to be dealt with, and therefore simplified formulæ are generally used which are more or less approximations. In practice, then, for approximate calculations it is commonly assumed that, above the critical velocity, the total resistance is proportional to the surface of contact and to the square of the average velocity of flow, and a variation in a coefficient is used to accommodate the approximate formulæ to the conditions of flow. At the same time a good knowledge of the general laws relating to the flow of fluids and the resistance offered to the flow which have been discussed is of fundamental importance to anyone desirous of calculating or judging the effect of different working conditions.

Thus, for the flow of water,

If a=area of section of pipe, sq. ft.
q=perimeter in contact with fluid, ft.
$\frac{a}{q}=m$=hydraulic mean depth, ft.
l=length of pipe, ft.
v=mean velocity of flow, ft. per sec.
δP=fall of pressure in length l, lbs. per sq. ft.
ρ=density of water, lbs. per cu. ft.
y=loss of head, ft. of water.

Then, $\delta P\times a=f^1ql\frac{v^2}{2g}$ is commonly assumed to represent the law of resistance above the critical velocity,

or,
$$\frac{\delta P}{\rho}=y=f\frac{l}{m}\frac{v^2}{2g} \quad \ldots\ldots\ldots \quad (18)$$

When the motion through a pipe is everywhere "stream line," that is, below the lower critical velocity, it can be shown that,

$$*\delta P=\frac{32\mu lv}{d^2} \quad \ldots\ldots\ldots \quad (19)$$

Above the critical velocity the value of n in equation 5, p. 67, is generally somewhat less than 2, and therefore in using the approximate equation 18 it may be expected that the coefficient f will vary with the diameter of the pipe, the velocity of flow, the roughness of the pipe, and the temperature of the water. To allow for the influence of the diameter

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* For proof see Gibson's *Hydraulics and its Applications.*

Darcy deduced from his experiments on the flow of water through cast-iron pipes from ·042 ft. to 1·67 ft. diameter,

$$y=\cdot005\left(1+\frac{1}{12d}\right)\frac{l}{m}\frac{v^2}{64\cdot4}\left\{\begin{matrix}\text{for clean and bare}\\ \text{metal surfaces}\end{matrix}\right\} \quad . \quad . \quad . \quad (20)$$

d being the diameter in feet.

$$\text{and, } y=\cdot01\left(1+\frac{1}{12d}\right)\frac{l}{m}\frac{v^2}{64\cdot4}\left\{\begin{matrix}\text{for old and incrusted}\\ \text{metal surfaces.}\end{matrix}\right\} \quad . \quad (21)$$

The experiments referred to on p. 68, made by J. G. Mair on the flow of water through a brass pipe 1½ in. diameter and 25 ft. long gave results which agreed closely with the equation,

$$y=\frac{b}{d^{1.2}}\frac{v^{1\cdot795}}{64\cdot4}l \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

where b had the following values :—

Water temperature °F.	57	70	80	90	100	120	130	160
Values of b . . .	·0178	·0169	·0166	·0157	·0151	·0147	·0145	·0133

The resistance, being proportional to the value of b, was therefore about 25% less at 160° F. than at 57° F.

In practice it is probably quite as good to make use of the simple equation 18, p. 82, and to adopt a suitable value of f. Figs. 23 and 23A* represent values of f for water-flow, Fig. 23 for old cast iron and also for cleaned cast-iron pipes, and Fig. 23A for new cast-iron pipes.

Equation 18 may also be considered to hold approximately for the flow of a gas through a tube or pipe at nearly constant temperature and pressure.

Thus, $$y=\frac{\delta P}{\rho}=f\frac{l}{m}\frac{v^2}{2g}$$

Now $$\rho=\frac{1}{V}=\frac{P}{C\tau}$$

where P, V, and τ are the absolute values of pressure, volume per unit mass of gas, and temperature respectively.

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* Gibson's *Hydraulics and its Applications.*

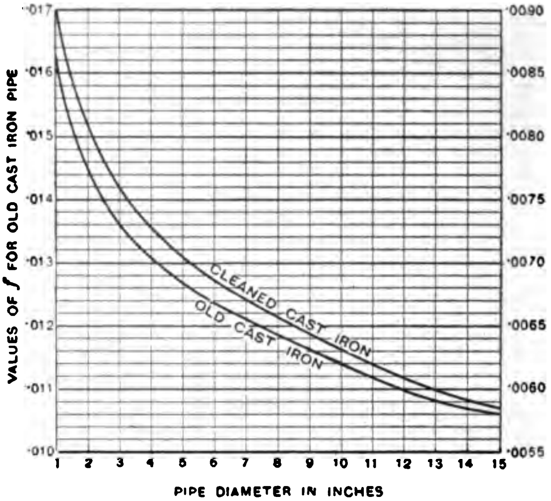

Fig. 23.—Relation between coefficient *f* and pipe diameter.

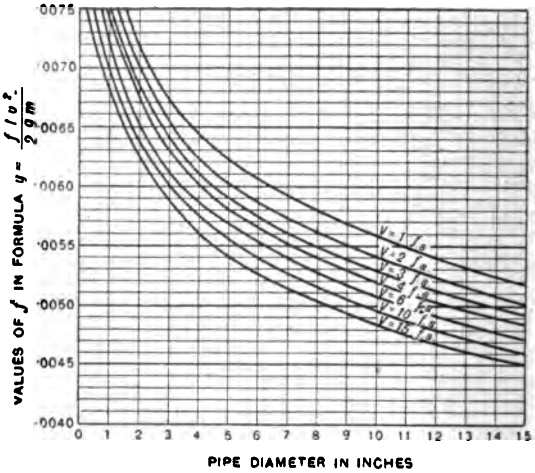

Fig. 23a.—Relation between coefficient *f* and pipe diameter for new cast-iron pipes.

$$\therefore \qquad \delta \mathrm{P}=f\frac{\mathrm{P}}{\mathrm{C}\tau}\frac{l}{m}\frac{v^2}{2g}$$

or, since $\rho v=\frac{w}{a}$, where w is the weight flowing per unit of time,

$$\text{then,} \qquad v^2=\left(\frac{w}{a}\right)^2\frac{1}{\rho^2}=\left(\frac{w}{a}\right)^2\left(\frac{\mathrm{C}\tau}{\mathrm{P}}\right)^2$$

$$\text{and,} \qquad \delta \mathrm{P}=\frac{f}{2g}\frac{\mathrm{C}\tau}{\mathrm{P}}\frac{l}{m}\left(\frac{w}{a}\right)^2$$

$$\text{or,} \qquad \frac{\delta \mathrm{P}\times \mathrm{P}}{\tau}=\frac{f\mathrm{C}}{2g}\frac{l}{m}\left(\frac{w}{a}\right)^2 \quad \ldots\ldots\ldots\ldots \quad (23)$$

If the units are lbs., ft., sec. °F., then $g=32{\cdot}2$ and $\mathrm{C}=53{\cdot}2$. It will be noted that equation 23 is of the same form as equation 8, p. 68, when $n=2$.

From experiments by Riedler, Gutermuth, and Stockalper, Professor Unwin* deduced the following values of f for air.

Diameter of pipe, feet .	·492	·656	·980
Value of f	·00449	·00377	·00290

Mr. H. A. Humphrey† gives the value of $f=\cdot00416$ for a pipe 2·5 ft. diameter, 5493 yds. long, conveying air at about atmospheric temperature. The pipe had one valve, nine drainage boxes, and ninety-five other special connections in the length under test.

Such an expression as 23 is not strictly applicable to the flow of gas through a boiler tube, for the reason that the temperature of the gas usually falls rapidly along the tube and the average velocity of the gas varies accordingly. There is thus a decrease of momentum of the flowing gas under such conditions which assists in overcoming the resistance to the flow. For approximate calculations, however, it may be assumed that equation 23 may be used even for such conditions, taking P and τ to represent the average absolute pressure and temperature respectively, but, of course, the coefficient f would still be under the influence of the changes of momentum along the tube. This point is discussed further on p. 171, and various values of f are deduced in Chapter IV applicable to such conditions.

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* *Hydraulics.*

† *Proc. Inst. Civil Engs.*, Vol. CXCII, 1912–13, Part II.

So far no account has been taken of the pressure required to create the kinetic energy of flow into the pipe or tube, or of the various losses of energy at the inlet to and at the outlet from the tube, and at bends, elbows, valves, etc.

Discussing briefly Bernoulli's theory* with respect to a perfect (i.e. non-viscous) incompressible fluid which is neither receiving nor losing energy, the relations between the kinetic energy $\left(\frac{v^2}{2g}\right)$, the pressure energy $\left(\frac{P}{\rho}\right)$, and the potential energy or height (Z) is expressed by :—

$$\frac{v^2}{2g}+\frac{P}{\rho}+Z=\text{constant} \quad . \; . \; . \; . \; . \; . \; . \quad (24)$$

Thus, if the height Z is constant an increase in the velocity v_1 to v_2 requires a corresponding difference of pressure (P_1-P_2) to produce it, or stated generally when there is no loss of energy :—

$$\frac{v_1^2}{2g}+\frac{P_1}{\rho_1}+Z_1=\frac{v_2^2}{2g}+\frac{P_2}{\rho_2}+Z_2 \quad . \; . \; . \; . \; . \; . \quad (25)$$

This equation may be applied without sensible error to a non-viscous compressible fluid when (P_1-P_2) is small compared with P_1 or P_2, which is usually the case in boilers and similar apparatus. In any case equation 24 applies generally in the form $\frac{v^2}{2g}+\int\frac{dP}{\rho}+Z=\text{constant}$ . . . . (26)

In ordinary calculations it is usual to take v as the average velocity in the pipe. Strictly speaking the values of v referred to in equations 24 to 26 should be the square root of the mean squares of the velocity over the section.

Losses of Energy at Inlet and Outlet.—It is well known that any sudden contractions or expansions in the path of a flowing fluid cause a loss of energy. With a sharp-edged inlet such as that shown in Fig. 24 (*a*), a fluid like water flows in-the manner indicated, giving a stream of contracted area at c, called the *vena-contracta*, and then suddenly expanding to fill the area of the pipe, thereby creating eddying motion in the fluid.

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* For a fuller account and proof refer to Gibson's *Hydraulics and its Applications*.

If v_c is the velocity at the section c, with area c
v_a ,, ,, ,, pipe section a, area a.

Then the loss of energy per unit mass of fluid is*

$$\frac{(v_c-v_a)^2}{2g}=\frac{v_a^2}{2g}\left\{\left(\frac{a}{c}\right)-1\right\}^2 \quad . \ . \ . \ . \ (27)$$

A similar expression also holds for the loss of energy at the inlet of a re-entrant pipe or tube represented in Fig. 24*b*, which shows approximately the conditions at the entrance to a condenser tube when not rounded at the inlet end.

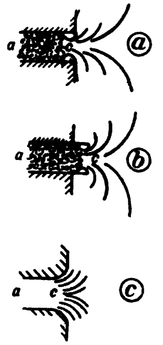

Fig. 24.—Flow of fluid into a tube or pipe.

For the case of Fig. 24 (a), $\frac{a}{c}$ may be taken at 1·685.

$$\text{Then, Energy loss}=\cdot 47\frac{v_a^2}{2g} \text{ per unit mass of fluid} \ . \ . \ . \ (28)$$

For the case of Fig. 24 (b), $\frac{a}{c}=2$

$$\text{and, Energy loss}=\frac{v_a^2}{2g} \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ . \ (29)$$

With the bell-mouthed inlet shown in Fig. 24 (c) having an entrance approximately the same in shape as the natural stream lines, the only loss of head is that due to the viscosity. In this case the loss of energy is, for water—

$$\text{Energy loss}=\cdot 05\frac{v_a^2}{2g} \text{ per unit mass} \ . \ . \ . \ . \ . \ . \ (30)$$

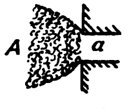

Fig. 25.—Flow of fluid from a plain tube.

At the outlet from an ordinary tube practically the whole of the kinetic energy in the flowing fluid is wasted.

If the area of the stream in Fig. 25 after leaving the tube is A

$$\text{Then, Energy loss at outlet}=\frac{v_a^2}{2g}\left\{1-\left(\frac{a}{A}\right)^2\right\}$$

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* For proof refer to Gibson's *Hydraulics.*

Usually $\frac{a}{A}$ is so small as to be practically negligible and the expression becomes,

$$\text{Energy loss at outlet} = \frac{v_a^2}{2g} \quad . \quad . \quad . \quad . \quad . \quad . \quad (31)$$

These equations 28 to 31 may also be applied to the flow of a compressible fluid when the losses of energy are such as occur, for instance, in the flow of gases through boiler tubes.

Experiments* on the flow of water through an expanding circular nozzle or outlet seem to show that the energy losses for all angles of divergence between 2° and about 5° may be taken at about,

$$\cdot 2\frac{v_a^2}{2g} \text{ per unit mass of water} \quad . \quad . \quad . \quad . \quad . \quad (32)$$

but for a greater divergence than this the energy loss rapidly increases with an increase of angle.

For the experiments on the flow of air made by the author, discussed on p. 182, the results indicate that about ·4 of the kinetic energy in the air at the tube exit can be converted into pressure energy by means of a divergent nozzle so long as the total angle of divergence is not greater than about 16°, whereas without an expanding nozzle at the outlet none of the kinetic energy was recovered as pressure energy. It may be stated that the large increase in the losses of energy at the outlet when the angle of divergence is made too great is mostly due to the eddying motions induced near the walls of the diverging outlet.

A comparison with Messrs. Heenan and Gilbert's† experiments on expanding chimneys on the delivery side of fans shows a fairly good agreement, i.e. about 0·4 of the kinetic energy of flow at the throat being recoverable as pressure energy.

In all these cases the axial kinetic energy per pound was calculated as $\frac{v_m^2}{2g}$, where v_m was the mean axial velocity determined by dividing the total volume of discharge by the total area available for flow.

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* Gibson's *Hydraulics*.

† *Proc. Inst. Civil Engs.*, Vol. CXXIII, p. 272.

Losses at Bends and Elbows.—Whenever the direction of motion of a stream is changed abruptly, as at a sharp bend or elbow, a loss of head occurs, apparently due to the formation of a *vena-contracta* on passing the elbow and the subsequent re-enlargement and shock which then takes place. The character of the flow is exhibited in Fig. 26.

According to various experimenters the loss of head due to the flow of water round a smooth bend in a pipe appears to be a minimum when the radius of curvature of the bend, R_0, is from 5 to 7·5 times the radius r of the pipe, increasing slightly up to the value R_0 about $13r$.

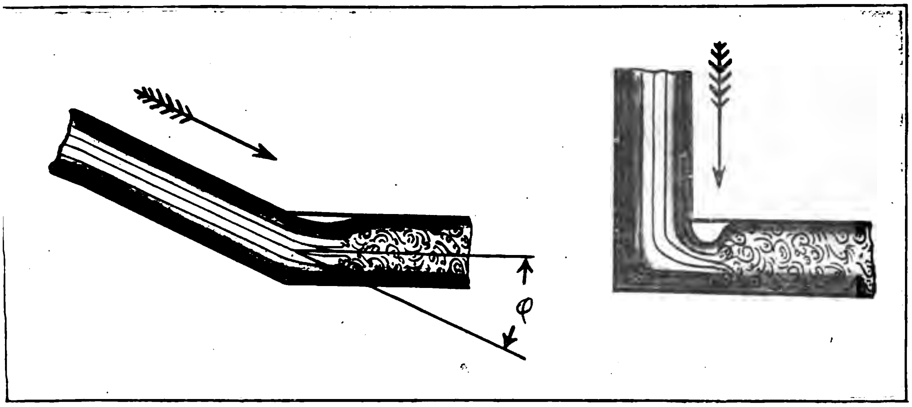

FIG. 26.—Flow of water at sharp bends.

Alexander* concludes that the loss of head due to a bend of radius $R_0 = 5r$ is equivalent to that offered by a straight length of pipe equal to $3{\cdot}38l$, where l is the length of the curved portion of the pipe.

Experiments made at the Yorkshire College† on an easy right-angled bend showed a resistance equivalent to a straight pipe 10 to 15 diameters in length, whilst for a sharp right angle bend or elbow the equivalent length was 30 to 36 diameters.

Experiments by Brightmore‡ indicate that for all curves of the best radius for all diameters of cast-iron pipes, and for all velocities, the additional loss of head due to the curvature in a right angle bend is equal to $\cdot 3\frac{v^2}{2g}$ ft. of water.

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* *Proc. Inst. Civil Engs.*, Vol. CLIX, p. 341.

† *Engineering*, Sept. 25, 1896, p. 390.

‡ *Proc. Inst. Civil Engs.*, Vol. CLXIX, p. 315.

Very little information is available with respect to the loss of head in a compressible fluid flowing round a bend or elbow, but in a general way the losses are likely to be of much the same relative order of magnitude as for water.

Losses at Valves, Cocks, etc.—The following experimental results for water are due to Weisbach. With a diaphragm of area a leading from a reservoir of large area into a pipe of area A, as in Fig. 27, taking the loss of head to be,

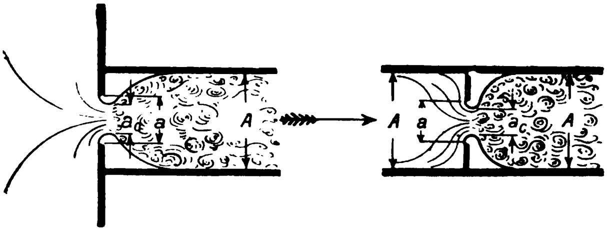

FIG. 27.—Flow of fluids in a tube through a diaphragm.

$$\text{Loss} = F\frac{v^2}{2g} \text{ feet of water,}$$

where v is the velocity in the pipe in feet per second and $g = 32 \cdot 2$, the following values are deduced :—

$\frac{a}{A}$	·1	·2	·4	·6	·8	1·0
F	231·7	50·99	9·612	3·077	1·169	0·48

With a diaphragm inserted in a pipe of area A, as in Fig. 27, the following values of F were obtained :—

$\frac{a}{A}$	·1	·2	·4	·6	·8	1·0
F	225·9	47·77	7·80	1·796	0·29	0

For a sluice or gate valve of rectangular section of depth D and opening of sluice *d*, as in Fig. 28 (*a*), F has the following values :—

$\frac{d}{D}$	·1	·2	·4	·6	·8	1·0
F	193	44·5	8·12	2·08	0·39	0

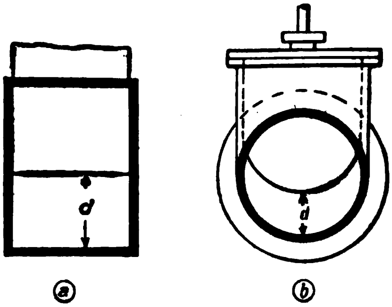

Fig. 28.—Opening of gate valve.

For a sluice or gate valve of circular section, where D is the pipe diameter and *d* the height of opening, as in Fig. 28 (*b*), F has the following values :—

$\frac{d}{D}$	$\frac{1}{8}$	$\frac{1}{4}$	$\frac{1}{2}$	$\frac{3}{4}$	1
F	97·8	17·0	2·06	0·26	0

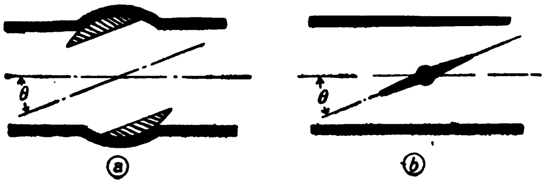

Fig. 29.—Opening of cock or valve.

For a cock in a cylindrical pipe, *a* the area through the cock, A area of pipe, and θ the angle through which the valve is turned, see Fig. 29 (*a*), the following are the values of F :—

θ	10°	20°	30°	40°	50°	60°	82°
$\frac{a}{A}$	·85	·69	·535	·385	·25	·14	cock closed
F	·29	1·56	5·47	17·3	52·6	206	—

For a throttle valve in a cylindrical pipe (Fig. 29 (b)), the following values of F are applicable :—

θ	10°	20°	30°	40°	50°	60°	70°
F	·52	1·54	3·91	10·8	32·6	118	751

Experiments on a gate valve for a 24-in. pipe* have given values of F about 50% greater than those of Weisbach, down to a quarter of full gate, but the form of the valve casing has some influence on the resistance.

Flow of Steam through Pipes.—The resistance to the flow of steam through pipes, bends, and valves, seems to have received comparatively little attention from a scientific point of view. Probably this is partly due to the difficulty of securing reliable measurements of the resistance over a sufficent length of pipe, particularly when the steam is supplied to, or exhausted from, reciprocating engines. So far as the writer knows, the results of experiments so far available do not seem to justify some of the complicated formulæ that have sometimes been proposed, and therefore, in expressing the results of experiments, use will be made only of the simple hydraulic formulæ given on p. 82, that is,

$$y=f\frac{l}{m}\frac{v^2}{2g}.$$

This expression may be modified as follows :—

If δp=loss of pressure in lbs. per sq. in.
ρ=mean density of the steam, lbs. per cubic ft.

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* Kuichling, *Trans. Amer. Soc. Civil Engs.*, 1892, Vol. 26.

Then, $\delta p=\frac{y\rho}{144}$,

∴ $\delta p=\frac{f}{144}\,\frac{l}{m}\,\frac{v^2}{2g}\,\rho=\frac{f}{144}\,\frac{l}{m}\,\frac{(v\rho)^2}{2g}\,\frac{1}{\rho}.$

If w=flow in lbs. per sec.

a=area of pipe in sq. ft.

$$v\rho=\frac{w}{a}$$

and $\frac{1}{\rho}=\mathrm{V}_s$=volume per lb. steam, cubic ft.

$$\therefore \quad \delta p=\frac{f}{144\times 2\times g}\,\frac{l}{m}\left(\frac{w}{a}\right)^2\mathrm{V}_s \quad \text{. . . . .} \quad (33)$$

$g=32{\cdot}2$

$m=\frac{d}{4}$ for circular pipe.

Some experiments have been made by Professor R. C. Carpenter* on a 90-ft. length of pipe, 2 in. diameter, the flow of steam being measured by the drop of pressure through a ¾-in. outlet nozzle in one case and a ⅞-in. nozzle in another series. He also made a single experiment on a 95-ft. length of pipe, 3 in. diameter. From the tabulated data in the paper the writer has calculated the values of f and the average is tabulated in Table 10, p. 94.

A rather lengthy series of experiments made by M. Ledoux on long steam pipes is recorded in *Annales des Mines*, Vol. II, 1892. The most important data from these experiments are also tabulated in Professor Carpenter's paper and from these the writer has deduced the various values of f shown in Table 10. It would be noted that in Ledoux's experiments the value of f increased as the diameter of the pipe increased, a result contrary to the general equations 5 and 6 of p. 67. In all these experiments referred to in Table 10, the value of $\left(\frac{w}{a}\right)^2 \mathrm{V}_s$ had but little influence on the value of f.

General comparisons between the various values of f for water, air, and steam, would indicate that the character of the flowing fluid for any particular rate of flow has comparatively little influence on the coefficient of resistance *f*. As might be anticipated from equation 6, p. 67, the influence of viscosity and density together seems to have only a minor influence on the resistance above the critical velocity of flow.

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* *Trans. Amer. Soc. Mech. Engs.*, Vol. 20, p. 342.

*TABLE 10

No. of Experiments in Series.	Pipe diameter inches.	Pipe length ft.	Steam pressure at inlet, lbs. sq. in absolute.	Values of $\left(\frac{w}{a}\right)^2 V_s$	Average values of *f*.	Experimenter.
10	2	90	105	3500 to 5500	·0062	Carpenter.
1	3	95	80	26,600	·00572	„
36	1·85	995 665 332	45 to 85	1470 to 4690	·00431	Ledoux.
36	2·79	1000 665 334	44 to 85	1200 to 6690	·00528	„
20	2·95	1083 755 426	44 to 88	433 to 4680	·00587	„
36	3·94	1003 665 336	41 to 82	1570 to 4880	·00591	„

Although there is comparatively little experimental evidence of a reliable nature to show how the value of *f* varies with the velocity and density of steam or with the diameter of the pipe, the fundamental equations on p. 67 indicate to some extent what to expect. In general the higher the velocity of flow the more nearly does the resistance (drop of pressure) become proportional to $\frac{\rho v^2 l}{d}$, as is shown more definitely by equation 17, p. 76, and seeing that under ordinary conditions of practice the velocity of steam in pipes is commonly from 100 to 130 ft. per second, and in the short exhaust pipe between a steam turbine and the condenser higher velocities are usually adopted, no great error is likely to occur by taking *f* to be practically independent of the density and velocity of the steam, and nearly independent of the diameter of the pipe.

In addition to the resistance to the flow through a pipe a drop of pressure is required to create the kinetic energy of flow. This is perhaps of no particular importance except in the exhaust pipe of a steam turbine or in the exhaust ports of a reciprocating steam engine. In the turbine the steam usually leaves the last row of blades at a high velocity and the exhaust pipe should be designed so as to convert as much as possible of the kinetic energy of flow into pressure energy. Thus, for example, if the absolute pressure of the steam at exhaust is 2 in. of mercury and the velocity 700 ft. per second from the last set of blades, the conversion of, say, ·4 of the kinetic energy into pressure energy is equivalent to an increased vacuum of about ·15 in.

Resistance of Valves and Bends to the Flow of Steam.—Professor Carpenter also made some experiments on the resistance to the flow of steam through globe and gate valves and at right-angle elbows for pipes up to 2½ in. diameter. Assuming the resistance could be expressed by,

$$\delta p=\frac{f\rho v^2}{2\times 144\times g}=\frac{f}{2\times 144\times g}\left(\frac{w}{a}\right)^2 V_s \quad . \ . \ . \ . \ (34)$$

to the same units as on p. 93, the following values of *f* were deduced :—

Value of *f*,	
8·2	for globe valves full open.
·005	for gate ,, ,, ,, (Kennedy pattern)
5·6	for right-angle elbows (mean radius of elbow equal to outside diameter of pipe).

These values of *f* for the globe valves and elbows are much greater than are commonly accepted, but there is very little experimental data available on larger pipes. It would be noticed how small is the resistance of a gate valve compared with a globe valve.

Measurement of Resistance to Flow of Fluids.—The measurement of the resistance to the steady flow of a fluid through a pipe or system of pipes is a matter of importance in experimental work. To measure the difference of pressure between any two points in a tube, one obvious method which might be adopted would be to measure the pressure at both points, either by liquid columns or by means of pressure gauges, and to subtract the values obtained. If the two points were at the same level this difference of pressure would represent the resistance to the flow between the two points, but if a difference of level existed, and the fluid was dense, as, for instance, water, an allowance would first need to be made for the difference of level. In any case, such a method is somewhat crude and, under most conditions, is liable to considerable errors. A better method is illustrated in Fig. 30 in connection with the resistance to the flow of water through a surface condenser C, the water flowing at a uniform rate as when delivered by a centrifugal pump, entering at A and leaving at D. At both these points a special connection could be made as shown in the enlarged view, small holes in the pipe connecting with the annular pocket P. The idea of this arrangement is to ensure an average pressure at this section of the pipe, and it might be mentioned that no burrs of any sort should be left inside the pipe round the edges of the small holes. The area of the section of the pipes at A and D should be the same so as to have the same velocity of flow. For approximate results, however, it would be sufficient to have suitable cocks or connecting tubes screwed into the ordinary piping at A and D, taking care to have no portion projecting into the pipe.

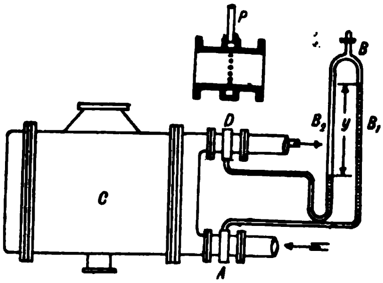

Fig. 30.—Measurement of resistance to flow of water through condenser.

The pressure at A is transmitted to the leg B_1 of the gauge, the connection being made by means of a stout rubber tube, or by a copper tube if the head is considerable. The pressure at D is similarly communicated to the other leg B_2 and the difference of level y represents the resistance to the flow between A and D. By the application of a hand pump (if necessary) at the cock B the air pressure in the common space above the water in B_1 and B_2 can usually be adjusted in order to retain the readings at suitable positions on the scales. The fact that the points A and D may be at different levels does not affect the value of y so long as the condenser tubes are full of water and the connecting tubes between the gauge B_1B_2 and points A and D are quite free from air pockets. It is perhaps necessary to mention that in estimating the level of the water in the legs of the gauge, the eye should be on a level with the surface and the reading taken at the bottom of the meniscus.

A mercury column may also be used to measure the resistance of the flow of water, arranged as shown in Fig. 31, the black portion representing mercury and the shaded portion water. Owing to the large density of mercury and the difficulty of reading the exact position of the surface of the columns of mercury this method is not suitable for measuring small differences of pressure. It should also be noted that the height y represents the pressure difference as a column of mercury minus a similar column of water in the other leg. Some care also needs to be taken when changing the rate of flow of the water, as a sudden change may produce a surging sufficient to cause the mercury column to leave the gauge tubes.

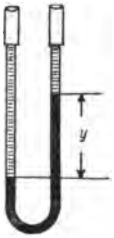

FIG. 31.—Mercury column to measure resistance to flow of a fluid.

To measure the resistance to the steady flow of a gas it is generally convenient to use rubber tube connections to a U-gauge of the form shown in Fig. 31, using water to indicate the resistance. In such an arrangement care sometimes needs to be taken not to have any loops in the rubber tubing where water vapour might condense and lodge, as this would obviously affect the reading of the gauge. If the difference of pressure happened to be very small the gauge might be made as represented in Fig. 32, where the connections would be made to the two ends of the inclined tube A. This tube A would be graduated

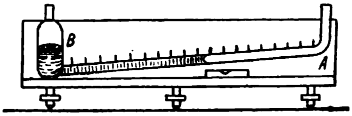

FIG. 32.—Measurement of small differences of gas pressure.

when fixed to a board at a convenient angle, and supported on a levelling base. The tube A is connected to the small vessel B, which is filled with water up to a zero mark, the water in the tube being then at the zero mark on the scale when no difference of pressure is being recorded.

A common arrangement of manometer or U-gauge is to bend a clean glass tube of about $\frac{3}{8}$-in. bore to the desired form and clip this to a hard wood board, after pasting a paper scale on the board. In this case, the scale should not be marked on the paper until it has been pasted on the board and become thoroughly dry, and it may afterwards be varnished over to keep it clean. Under some circumstances it might be convenient to arrange a sliding vernier on each leg of the gauge, by which means more accurate readings are obtainable, but if small pulsations occur in the levels the vernier scale is not of much service.

If the difference of level y is comparatively small and it is found necessary to measure it accurately a cathetometer may be used for this purpose. This instrument consists of a telescope provided with cross-wires in the eyepiece, and mounted on a vertical pillar. The telescope can be raised or lowered, while its axis remains horizontal, and adjusted so that the image of the surfaces of the column of liquid coincides with the intersection of the cross wires. The vertical distance through which the telescope is raised or lowered can be accurately observed on a scale on the stand or pillar.

An elaborate but accurate arrangement for measuring small steady differences of pressure is sold by the Cambridge Scientific Instrument Company, the principle of which is shown in Fig. 33. The micrometer screw M with graduated head indicates the difference of level between the water or oil in the small vessels A and B.

The above-mentioned company also manufacture the Chattock tilting micro-manometer for measuring small differences of pressure very accurately. The principle of this gauge is that of the U-tube with the two limbs widely apart, and the difference of pressure on the surfaces of the water in the two limbs of the tube is measured by tilting the gauge through a very small angle so that there is no displacement of the water along the tube. The frame which carries the glasswork rests upon three hardened steel points in the lower part. One of these points forms the extremity of the axis of the spindle and wheel by means of which the tilting is measured. A difference of pressure in the two vessels forming the extremities of the U-tube causes a displacement of the surface which is kept in coincidence with the cross-wire of a microscope by rotating the hand wheel. The movement of the gauge is read off on the hand wheel and the pressure head is thus determined. The scale on the wheel rim has 100 divisions and a movement of the wheel of one-tenth of a scale division can be detected by the microscope, and this enables the pressure difference to be observed to ·00006 in. of water. It will therefore be recognised that such an instrument is not commonly necessary for ordinary practical conditions in measuring differences of pressure, and would only be serviceable where an extreme degree of accuracy is essential.

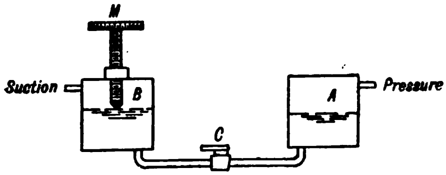

Fig. 33.—Micrometer arrangement for measuring small differences of gas pressure.

Circulation in Boilers.—The proper circulation of the water in a boiler is necessary to prevent overheating of the metal, particularly so when a boiler is heavily forced. Under ordinary conditions of operation the natural circulation or convection currents set up by the heating of the water and the generation of steam is sufficient for the purpose. Referring more particularly to the circulation in water-tube boilers, however, it may be said that when steam is being generated the circulation is rapid and takes place in a definite manner, that is, the water flows down the downcomer tubes and the mixture of water and generated steam ascends through the heated upcomer tubes. The following experiment carried out by C. A. Matthey* illustrates the phenomena and cause of the rapid circulation in a water-tube boiler when steam is being generated.

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* *Trans. Inst. Engs. and Shipbds. in Scotland,* Vol. XLI, pp. 147–150.

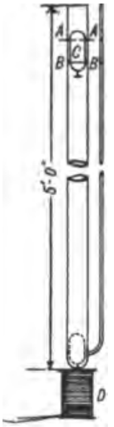

Fig. 34. Matthey's apparatus to show the effect of a rising bubble on the level of water in a tube.

A vertical glass tube, 2 in. diameter and 5 ft. high, connected at the bottom with another glass tube of the same height but only one-eighth of an inch in diameter, was placed vertically upon an electro-magnet D, as shown in Fig. 34. The tubes were nearly filled with water, and a glass bubble a little smaller than the large tube floated within it with the water level at BB in both tubes. A small piece of soft iron was attached to the bottom of the bubble so that when the bubble was pushed down to the bottom of the tube it could be held in position there by the electro-magnet, or released suddenly on breaking the current. When the bubble was fully immersed and prevented from rising the water level became AA in both legs due to the displacement of the bubble. On releasing the bubble so that it was free to rise to the surface the water level in the small gauge tube began to fall from A towards B and reached B as soon as the bubble had attained its maximum velocity of ascent. Thus the bubble, when free to ascend under the action of the impressed upward forces, reduced the pressure at the bottom below that corresponding to the height of the column of water in the tube containing the bubble. The reasons for this are not very evident without further discussion. It is well known that when a body is immersed in a fluid the upward force on the body equals the weight of fluid displaced by the body. When the bubble was held stationary at the bottom of the tube this upward force was balanced by the attraction of the electro-magnet and the water level was the same in both legs, but as soon as released, the upward force was free to accelerate the bubble which thus acquired some particular velocity where the resistance to the motion of the bubble through the water balanced the upward thrust on the bubble, and then the velocity would be uniform until the surface was reached. It follows, therefore, that the reduction of the pressure at the bottom of the tube was due to the frictional forces between the moving bubble and the water, and had the same effect as a reduction of the specific gravity of the water in the tube.

An action similar to this occurs when steam bubbles are free to rise to the surface in a boiler tube. Let the tube U in Fig. 35 (*a*) and (*b*) represent the heated tubes in which steam is being generated, and supposing first there is a watertight division wall A separating the boiler drum into two parts, as in Fig. 35 (*a*), then the presence of the ascending bubbles of steam would cause the water level on the left of A to be higher than that on the right of A, for the reasons explained in connection with the experiment represented in Fig. 34. If the division wall A were taken away the conditions would be as represented in Fig. 35 (*b*), which approximate to boiler conditions, and the difference of level shown in Fig. 35 (*a*) is now to some extent available for causing circulation in the direction shown.

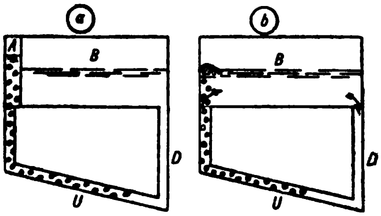

Fig. 35.—Circulation in boiler tubes.

The earlier the steam is generated in the tube U, all other conditions the same, the greater would be the rate of circulation, because the quantity of steam bubbles in the tube U would thereby be increased. Thus, heating the downcomers D may promote circulation by enabling steam to be produced at an early period of the circuit. Mr. Yarrow's experiments have shown that heating the downcomers as well as the upcomers does actually increase the rate of circulation under certain conditions.

If the bore of the tube is so small that the steam and water form alternate plugs, the circulation is not necessarily the same as when the bubbles can rise freely, and the circulation is then apt to be spasmodic in character.

The rate of circulation in any boiler of the water-tube type will be some function of the force available for circulation, and under certain circumstances this force can be measured approximately by means of a water column suitably arranged. Referring to the sketch, Fig. 36, S is the top drum connected to the bottom drum W by the downcomer D and the upcomers T, the circulation being in the direction shown.* The gauge glass G indicates the level of the water in the drum S, and the gauge B connects the bottom water drum with the steam space in the top drum. If the water in the column B is at the same temperature and density as that in D, then the difference of level y is a measure of the force causing circulation under the particular conditions of working. The temperature of the water in B would be practically the same as that in D if the water in B were blown off just before a reading is required and then allowed to fill with water from the drum W; otherwise, an allowance might need to be made for the difference of density between the water in B and D. It is evident that if any appreciable quantity of steam were present or were being generated in the downcomers, the average density in the downcomers could not be estimated and the arrangement might not then be as reliable for giving a correct measure of the force causing circulation. Under working conditions it is generally found that the level of the water in the gauge glasses oscillates somewhat and an average position would have to be estimated. It is probable that the rate of circulation of the water in a water-tube boiler is not far from being proportional to the square root of the difference of level y.

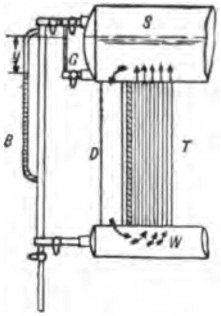

FIG. 36.—Measurement of force causing circulation in water-tube boilers.

Such an arrangement as that shown in Fig. 36 was used by Thornycroft† in connection with experiments on models of water tube boilers. In one model the upcomers discharged above the water level in the drum and in another below the water level. Experiments were made at two different steam pressures and at rates of evaporation varying between 3 lb.

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* It is to be noted that some of the tubes T furthest from the fire may also act as downcomers as well as the special downcomer D.

† *Trans. Inst. Naval Arch.*, July, 1894.

to over 15 lb. per hour per square foot of heating surface, and the reduction of pressure in the lower chamber (measured as height y, Fig. 36) was from $2\frac{1}{2}$ to 3 times greater in the boiler where all the upcomers delivered below the water level in the drum than in the one in which delivery took place above the water level. It was therefore calculated that the velocity of the water in the downcomers would be from 60 to 70% greater in the former case than in the latter.*

Blechynden† carried out similar experiments on a full-sized water tube boiler, having a total heating surface of 2445 sq. ft., and a grate surface of 37 sq. ft. The small generating tubes were 1 in. external diameter, and the upper and lower drums were also connected by eight stay tubes, $1\frac{1}{4}$ in. internal diameter, placed outside the casing. Also, to each bottom chamber was connected a 5-in. internal diameter downcomer which were arranged so that they could be shut off. The feed water could be delivered either into the upper drum or into the two lower drums. Special gauge glasses were fitted in some of the outer rows of tubes to allow the direction of the current in these tubes to be observed, and it was found that the current was downwards in the outer row when the water level was above their upper ends. The reduction of pressure in the lower drums was measured in the manner described in connection with Fig. 36.

So far as feeding the water was concerned, three sets of experiments were made: (I) with the feed in the upper drum; (II) with the feed in the lower drums; and (III) with the feed shut off entirely; the last experiments, however, could only be made for short periods of 3 or 4 minutes.

The actual results of loss of pressure in this lower chamber during a series of experiments are shown in Fig. 37, plotted on a base of evaporation per square foot of heating surface. Curve No. 1 shows the results of four experiments with the large downcomers open and the feed entering at the bottom. No. 2 gives the results of two experiments under similar conditions but with the feed entering the top drum, whilst No. 3 gives the results obtained with the feed shut off for short periods. Curve No. 4 refers to the values obtained with the large downcomers shut off and the feed entering at the top. The difference between curves No. 2 and No. 4 is therefore due to the effect of the large outside downcomers.

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* *Proc. Inst. Civil Engs.*, Vol. CXXXVII, 1898–99, Part III, p. 177.

† *ibid.*, pp. 178 to 181.

There is always some uncertainty as to how many of the generator tubes are acting as upcomers and how many as downcomers, and therefore it is hardly possible to obtain a definite measure of the rate of circulation in an actual boiler.

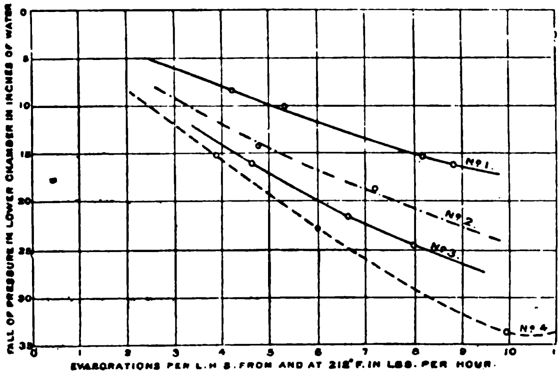

FIG. 37.—Relation between evaporation and the pressure drop in lower chamber of a water-tube boiler.

Professor Watkinson* used a similar arrangement of gauges to determine the rate of circulation in a single downcomer and single upcomer model of a water tube boiler evaporating water at atmospheric pressure. The upcomer was $1\frac{1}{8}$ in. diameter and heated by a special gas burner; different diameters of downcomers were used. Also the upcomer could be arranged to discharge the steam and water either above or below the water level in the top drum as desired. The values in Table 11 illustrate some of the results obtained when evaporating 13 lb. of water per hour at atmospheric pressure; the evaporation per hour per square foot of heating surface was about 10·8 lb.

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* "Circulation in Water-Tube Boilers," *Inst. Naval Arch.*, 1896.

TABLE 11

Area of downcomer, sq. in.	Area of upcomer. / Area of downcomer.	Depression of column y inches.		Weight of circulating water. / Weight of feed water.	
		Discharge above water level.	Discharge below water level.	Discharge above water level.	Discharge below water level.
·087	11	21	18	40	37
·359	2·6	8¼	7¼	147	138
·994	0·95	3¼	3¼	242	242

The relation between the difference of level y and the rate of circulation was determined by detaching the upcomer and inserting various orifices on the connection from the bottom drum. Hot water was allowed to flow through at definite rates and the value of y observed. In all cases an allowance was made to compensate for the difference in density between the water in the boiler and that in the gauge connected to the bottom drum. It was found that the quantity of water flowing was proportional to the value of $\sqrt{y}$ for any given downcomer, where y is represented in Fig. 36.

Experimental work on heat transmission between flowing fluids generally involves measurements of the quantity of fluid flowing per unit of time. It is not proposed to discuss here the many possible methods which are available, and for information of this character reference might be made to the author's book on *The Testing of Motive Power Engines.*

CHAPTER IV

THE TRANSMISSION OF HEAT BY CONVECTION

When a fluid flows through a tube or parallel channel at velocities below the "critical" value the fluid flows without eddying motions. Under these conditions heat can only be transmitted to or from the fluid by conduction, natural diffusion, and radiation. The conductivity of fluids and the rate of natural diffusion are so low that very little heat can be transmitted by these means. The emission and absorption of heat by fluids is also comparatively ineffective under ordinary conditions. Therefore, the rapid transmission of heat between a fluid and a surface can only be effected through the convective action which occurs with the eddying flow above the critical velocity, that is, by some of the molecules in the rapidly moving fluid jumping into the slower moving regions at or near the surface of contact, and a corresponding number jumping back into the rapidly moving fluid. Particularly with gases these interchanges of molecules in the flowing fluid involve a corresponding interchange of axial momentum which, at any rate for a gas, constitutes the resistance offered to the flow of the fluid, as was demonstrated on pp. 61 to 63. Therefore there is every reason to believe that the transmission of heat from or to a fluid (other than by radiation) and the resistance to the flow of the fluid are principally due to one and the same phenomena of molecular movement. Considerations such as these led Professor Osborne Reynolds to his theory of heat transmission between a fluid and a surface discussed on pp. 111 to 115.

The resistance offered to heat-flow from a fluid on one side of a plate or tube to another fluid on the other side is due to the various resistances in series, which may be enumerated as follows :—

1. The resistance between the body of hot fluid and the surface of contact, which probably includes a small surface resistance between the fluid and the surface.
2. The resistance offered by any deposit of scale, soot, dirt, oil or oxydised metal, including probably a surface resistance between the deposit and the metal.
3. The resistance of the metal plate or tube.
4. Similar resistances to 2 and 1 between the other metal surface and the other fluid.

The total resistance to heat-flow is the sum of the separate resistances taken in series, and to overcome these resistances differences of temperature are necessary proportional to the resistances offered and to the amount of heat transmitted per unit of time.

Figs. 38 (*a*) and (*b*) have been drawn to represent the approximate distribution of temperature from the gas side to the water side of a boiler plate working under ordinary conditions. Temperatures are set off vertically and distances from gas to water measured horizontally. In Fig. 38 (*a*) the metal surfaces are supposed to be clean, whereas in (*b*) there is represented a deposit of soot, dirt, or oxide on the gas side and a deposit of scale on the water side.

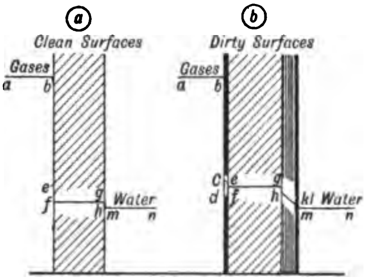

FIG. 38.—Drop of temperature from gas to water in boilers.

The small temperature differences *ef* and *gh* represent respectively the probable drop of temperature between the gas and the surface of contact and between the other surface of contact and the water, although the actual magnitude of these values is unknown. In Fig. 38 (*b*) there is also represented the fall of temperature through the soot, dirt, etc., on the gas side of the plate, and similarly through the scale on the water side, with, of course, the probable additional surface resistances causing the falls of temperature *cd*, *ef*, *gh*, and *kl*. From these diagrams it will readily be realised how small is the drop of temperature through the metal and from the metal to the water compared with that from the gas to the surface.

The diagrams in Figs. 39 (*a*) and (*b*) have been drawn to represent the temperature conditions in a surface condenser from the steam to the cooling water with about the same heat-flow as in the boiler, Fig. 39 (*a*) for clean metal surfaces and (*b*) for surfaces covered with deposits of oil, dirt, or scale. It is seen that the difference of temperature between the steam and the metal surface is very small compared with that between a gas and a surface, and therefore the difference of temperature between the steam and the water in a surface condenser is very much less than that between the gas and water in a boiler for the same flow of heat.

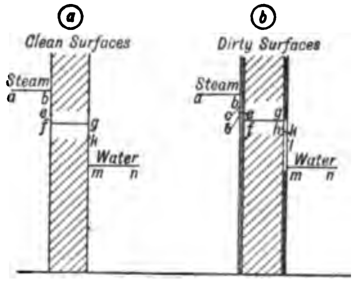

Fig. 39.—Drop of temperature from steam to water in surface condensers.

$$\text{Let } h_1 = \begin{cases} \text{heat-flow per unit surface per unit time per deg.} \\ \text{temperature difference between the flowing fluid} \\ \text{(gas or steam) and the metal surface,} \end{cases}$$

h_m=heat-flow, to the same units, through the metal,

h_2= ,, ,, ,, between the metal surface and the flowing water,

Then, $\frac{1}{h_1}$, $\frac{1}{h_m}$, $\frac{1}{h_2}$ may be taken to represent the respective resistances to the flow of heat. The total resistance therefore becomes,

$$\frac{1}{h}=\frac{1}{h_1}+\frac{1}{h_m}+\frac{1}{h_2} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \quad (1)$$

The values of $\frac{1}{h_1}$ and $\frac{1}{h_2}$ depend upon the flow of fluid past the surfaces, as will afterwards be shown; but taking for ordinary boiler conditions the relative values,

$$\frac{1}{h_1}=\frac{1}{\cdot 0026}=385,\ \frac{1}{h_2}=\frac{1}{\cdot 2}=5,\ \frac{1}{h_m}=\frac{1}{\cdot 17}=5\cdot 9,$$

where h is measured in B.Th.U. per sq. ft. per sec. per deg. Fahr. difference,

then, $\frac{1}{h}=385+5+5{\cdot}9=396.$

It is seen that the resistance of the metal and that between the metal and the water are both relatively small, and that a large proportionate reduction of these two resistances could not appreciably reduce the total resistance. For example, if $\frac{1}{h_2}$ and $\frac{1}{h_m}$ were both halved the total resistance would only be reduced from 396 to about 390.

It follows, therefore, that any large reduction in the value of $\frac{1}{h}$ could only be obtained by a reduction of the value of $\frac{1}{h_1}$, i.e. the resistance on the gas side of the plate.

In the case of the surface condenser, however, taking as representative values $\frac{1}{h_1}=\frac{1}{\cdot 4}=2{\cdot}5$ on the steam side, $\frac{1}{h_m}=\frac{1}{7}=\cdot 14$, and $\frac{1}{h_2}=\frac{1}{\cdot 4}=2{\cdot}5$ on the water side, it is seen that $\frac{1}{h_2}$ is now a fairly large portion of the total resistance $\frac{1}{h}$, and therefore a large proportionate reduction of $\frac{1}{h_2}$ would have a large influence on $\frac{1}{h}$. If, however, the presence of air on the steam side increased $\frac{1}{h_1}$ from $\frac{1}{\cdot 4}=2{\cdot}5$ to $\frac{1}{\cdot 1}=10$, say, then the resistance on the steam and air side would become a large determining factor on the heat-flow, and changes of $\frac{1}{h_2}$ would have a correspondingly smaller influence.

The influence of a deposit of scale or dirt on the water side of the surface is to increase the resistance to heat-flow. As an example consider the resistance $\frac{1}{h_2}$ in a boiler to be increased by scale deposit from, say, $\frac{1}{\cdot 2}=5$ for clean plates, to $\frac{1}{\cdot 04}=25$, an increase of 500% on the water side. The total resistance

$\frac{1}{h}$ however, is only increased by about 5%, whereas, a similar increase of $\frac{1}{h_2}$ in a condenser would probably increase the total resistance $\frac{1}{h}$ by, say, 300%. Therefore a thin layer of scale or dirt on the water side of a boiler plate would have only a comparatively small influence on the rate of heat-flow from the gases to the water, but in a condenser its influence would probably be serious. It will be noted, however, that a deposit on the water side tends to raise the temperature of the metal, which may be very undesirable in boilers. It follows also that an increase of heat resistance on the gas side due to a deposit of a thin layer of soot, dirt, or oxidised metal would have a corresponding influence on the rate of heat-flow.

For the case of a tube of outside diameter r_1 and inside diameter r_0, equation 1, p. 108, is modified as follows. Assume the gas or steam to be on the outside and the water to be inside the tube, and that the rate of heat-flow h is calculated on the outside area of the tube. Then per degree difference of temperature and per unit length of tube.

Total flow of heat (gas or steam to water) $=h2\pi r_1$ per degree difference between the fluids.
From gas or steam to wall $=h_1 2\pi r_1$ per degree difference.
From metal to water $=h_2 2\pi r_0$ " "
Through metal (eqn. 3, p. 24) $= \dfrac{2\pi k}{\log_e \frac{r_1}{r_0}}$ " "

$$\text{Thus, } \left(\frac{1}{hr_1} = \frac{1}{h_1 r_1} + \frac{1}{k}\log_e\frac{r_1}{r_0} + \frac{1}{h_2 r_0}\right)$$

$$\text{or, } \frac{1}{h} = r_1\left(\frac{1}{h_1 r_1} + \frac{1}{k}\log_e\frac{r_1}{r_0} + \frac{1}{h_2 r_0}\right) \quad \ldots\ldots\ldots\ldots \quad (2)$$

If the gas or steam is inside the tube and the water outside

$$\text{then, } \frac{1}{h} = r_0\left(\frac{1}{h_1 r_0} + \frac{1}{k}\log_e\frac{r_1}{r_0} + \frac{1}{h_2 r_1}\right) \quad \ldots\ldots\ldots\ldots \quad (3)$$

where h is now calculated on the inside area of the tube. Unless the tube happens to be of very small bore and relatively thick walls there is not much difference between the results of equations 2 or 3, and equation 1, p. 108.

In recent years the theory of heat transmission first enunciated publicly by Professor Osborne Reynolds in 1874 has received considerable support by direct experimental evidence, and since the Reynolds law of heat transmission is now fairly well established, it is proposed to reprint, partly for instruction and partly for historical reasons, the original paper read by Professor Reynolds.

Reprinted from the Proceedings of the Literary and Philosophical Society of Manchester, 1874

" On the Extent and Action of the Heating Surface for Steam Boilers," by Professor Osborne Reynolds, M.A.

The rapidity with which heat will pass from one fluid to another through an intervening plate of metal is a matter of such practical importance that I need not apologise for introducing it here. Besides its practical value it also forms a subject of very great philosophical interest, being intimately connected with, if it does not form part of, molecular philosophy.

In addition to the great amount of empirical and practical knowledge which has been acquired from steam-boilers, the transmission of heat has been made the subject of direct inquiry by Newton, Dulong and Petit, Péclet, Joule, and Rankine, and considerable efforts have been made to reduce it to a system. But as yet the advance in this direction has not been very great; and the discrepancy in the results of the various experiments is such that one cannot avoid the conclusion that the circumstances of the problem have not been all taken into account.

Newton appears to have assumed that the rate at which heat is transmitted from a surface to a gas and vice versa is *ceteris paribus* directly proportional to the difference in temperature between the surface and the gas, whereas Dulong and Petit, followed by Péclet, came to the conclusion from their experiments that it followed altogether a different law.*

These philosophers do not seem to have advanced any theoretical reasons for the law which they have taken, but have deduced it entirely from their experiments, " à chercher par tâtonnement la loi que suivent ces résultats."†

In reducing these results, however, so many things had to be taken into account and so many assumptions have been made that it can hardly be a matter of surprise if they have been misled. And there is one assumption which upon the face of it seems to be contrary to general experience, this is, that the quantity of heat imparted by a given extent of surface to the adjacent fluid is independent of the motion of that fluid or of the nature of the surface ;* whereas the cooling effect of a wind compared with still air is so evident that it must cast doubt upon the truth of any hypothesis which does not take it into account.

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* *Traité de la Chaleur*, Péclet, Vol. I, p. 365.

† *Ib.*, p. 363.

In this paper I approach the problem in another manner from that in which it has been approached before. Starting with the laws recently discovered of the internal diffusion of fluids I have endeavoured to deduce from theoretical considerations the laws for the transmission of heat, and then verify these laws by experiment. In the latter respect I can only offer a few preliminary results ; which, however, seem to agree so well with general experience as to warrant a further investigation of the subject, to promote which is my object in bringing it forward in the present incomplete form.

The heat carried off by air or any fluid from a surface, apart from the effect of radiation, is proportional to the internal diffusion of the fluid at and near the surface, i.e. is proportional to the rate at which particles or molecules pass backwards and forwards from the surface to any given depth within the fluid, thus, if AB be the surface and *ab* an ideal line in the fluid parallel to AB, then the heat carried off from the surface in a given time will be proportional to the number of molecules which in that time pass from *ab* to AB—that is for a given difference of temperature between the fluid and the surface.

This assumption is fundamental to what I have to say, and is based on the molecular theory of fluids.

Now the rate of this diffusion has been shown from various considerations to depend on two things :—

1. The natural internal diffusion of the fluid when at rest.
2. The eddies caused by visible motion which mixes the fluid up and continually brings fresh particles into contact with the surface.

The first of these causes is independent of the velocity of the fluid, if it be a gas is independent of its density, so that it may be said to depend only on the nature of the fluid.†

The second cause, the effect of eddies, arises entirely from the motion of the fluid, and is proportional both to the density of the fluid, if gas, and the velocity with which it flows past the surface.

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* *Traité de la Chaleur,* Péclet, Vol. I, p. 383.

† Maxwell's *Theory of Heat,* Chap. XIX.

The combined effect of these two causes may be expressed in a formula as follows :—

$$H = At + B\rho vt, \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (4)$$

where t is the difference of temperature between the surface and the fluid, ρ is the density of the fluid, v its velocity, and A and B constants depending on the nature of the fluid, H being the heat transmitted per unit of the surface in a unit of time.

If therefore a fluid were forced along a fixed length of pipe which was maintained at a uniform temperature greater or less than the initial temperature of the gas we should expect the following results :—

1. Starting with a velocity zero, the gas would then acquire the same temperature as the tube. 2. As the velocity increased the temperature at which the gas would emerge would gradually diminish, rapidly at first, but in a decreasing ratio until it would become sensibly constant and independent of the velocity. The velocity after which the temperature of the emerging gas would be sensibly constant can only be found for each particular gas by experiment ; but it would seem reasonable to suppose that it would be the same as that at which the resistance offered by friction to the motion of the fluid would be sensibly proportional to the square of the velocity. It having been found both theoretically and by experiment that this resistance is connected with the diffusion of the gas by a formula :—

$$R = A^1 v + B^1 \rho v^2 \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (5)$$

And various considerations lead to the supposition that A and B in (4) are proportional to A^1 and B^1 in (5). The value of v which this gives is very small, and hence it follows that for considerable velocities the gas should emerge from the tube at a nearly constant temperature whatever may be its velocity.

This, as I am about to point out, is in accordance with what has been observed in tubular boilers as well as in more definite experiments.

In the locomotive the length of the boiler is limited by the length of tube necessary to cool the air from the fire down to a certain temperature say 500°. Now there does not seem to be any general rule in practice for determining this length, the length varying from 16 ft. to as little as 6, but whatever the proportions may be each engine furnishes a means of comparing the efficiency of the tubes for high and low velocities of the air through them. It has been a matter of surprise how completely the steam-producing power of a boiler appears to rise with the strength of blast or the work required from it. And as the boilers are as economical when working with a high blast as with a low, the air going up the chimney cannot have a much higher temperature in the one case than in the other. That it should be somewhat higher is strictly in accordance with the theory as stated above.

It must, however, be noticed that the foregoing conclusion is based on the assumption that the surface of the tube is kept at the same constant temperature, a condition which it is easy to see can hardly be fulfilled in practice.

The method by which this is usually attempted is by surrounding the tube on the outside with some fluid the temperature of which is kept constant by some natural means, such as boiling or freezing, for instance the tube is surrounded with boiling water. Now although it may be possible to keep the water at a constant temperature it does not at all follow that the tube will be kept at the same temperature; but, on the other hand, since heat has to pass from the water to the tube there must be a difference of temperature between them, and this difference will be proportional to the quantity of heat which has to pass. And again the heat will have to pass through the material of the tube, and the rate at which it will do this will depend on the difference of the temperatures at its two surfaces. Hence if air be forced through a tube surrounded with boiling water, the temperature of the inner surface of the tube will not be constant but will diminish with the quantity of heat carried off by the air. It may be imagined that the difference will not be great: a variety of experiments lead me to suppose that it is much greater than is generally supposed. It is obvious that if the previous conclusions be correct this difference would be diminished by keeping the water in motion, and the more rapid the motion the less would be the difference. Taking these things into consideration the following experiments may, I think, be looked upon, if not as conclusive evidence of the truth of the above reasoning yet as bearing directly upon it.

One end of a brass tube was connected with a reservoir of compressed air, the tube itself was immersed in boiling water, and the other end was connected with a small non-conducting chamber formed of concentric cylinders of paper with intervals between them in which was inserted the bulb of a thermometer. The air was then allowed to pass through the tube and paper chamber, the pressure in the reservoir being maintained by bellows and measured by a mercury gauge: the thermometer then indicated the temperature of the emerging air. One experiment gave the following results :—With the smallest possible pressure the thermometer rose to 96° F., and as the pressure increased fell, until with $\frac{1}{10}$ in. it was 87°, with $\frac{1}{4}$ inch it was 70°, with 1 in. it was 64°, with 2 in. 60° ; beyond this point the bellows would not raise the pressure.

It appears, therefore, (1) that the temperature of the air never rose to 212°, the temperature of the tube, even when moving slowest ; but this difference was clearly accounted for by the loss of heat in the chamber from radiation, the small quantity of air passing through it not being sufficient to maintain the full temperature, an effect which must obviously vanish as the velocity of the air increased ; (2) as the velocity increased the temperature diminished, at first rapidly and then in a more steady manner. The first diminution might be expected from the fact that the velocity was not as yet equal to that at which the resistance of friction is sensibly equal to the square of the velocity as previously explained. The steady diminution which continued when the velocity was greater was due to the cooling of the tube. This was proved to be the case, for at any stage of the operation the temperature of the emerging air could be slightly raised by increasing the heat under the water so as to make it boil faster and produce greater agitation in the water surrounding the tube. This experiment was repeated with several tubes of different lengths and characters, some of copper and some of brass, with practically the same results. I have not, however, as yet been able to complete the investigation, and I hope to be able before long to bring forward another communication before the Society.

I may state that should these conclusions be established, and the constant B for different fluids be determined, we should then be able to determine, as regards length and extent, the best proportion for the tubes and flues of boilers.

In further explanation of his theory Professor Reynolds contributed the following demonstration to the paper by Dr. Stanton referred to on p. 117.

"The motion of heat from the surface of the pipe follows the same laws as the motion of momentum to the surface, whether by conduction or convection (though not by radiation and absorption through the material, which unquestionably plays an important part in the so-called conduction of water).*

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* In the original paper T_0, D and w were the symbols used to represent the temperature of the pipe, density of water, and velocity respectively.

Let x be the direction of motion of the water, and let

r=radius of pipe.
l=length of pipe.
t=temperature of water, °C.
θ=temperature of pipe, °C.
ρ=density of water.
w=weight of water discharged.
v=velocity of water through the pipe.
p=pressure of water per unit area.
σ=viscosity of water$=(1+\cdot0336t+\cdot000221t^2)^{-1}$[*]

and let A, B, and n be constants depending upon the nature of the surface.

Then, above the critical velocity the loss of pressure is given by the equation :—

$$\frac{dp}{dx}=\frac{\sigma^{2-n}}{(2r)^{3-n}}v^n\frac{\mathrm{B}^n}{\mathrm{A}}\quad .\ .\ .\ \text{(see equation 6, p. 67).}$$

Writing this in the form,

$$\pi r^2\frac{dp}{dx}=\pi r^2\frac{g}{w}\frac{\sigma^{2-n}}{(2r)^{3-n}}v^{n-1}\frac{\mathrm{B}^n}{\mathrm{A}}\left(\frac{w}{g}v\right).$$

Then $\pi r^2\dfrac{dp}{dx}$ is the loss of momentum due to diffusion and convection and $\dfrac{wv}{g}$ is the momentum of the fluid, so that, according to the above theory, substituting the loss or gain of heat $w\dfrac{dt}{dx}$ for $\pi r^2\dfrac{dp}{dx}$, and the heat in the fluid above or below the wall temperature $w(\theta-t)$ for $\dfrac{w}{g}v$, the equation for the passage of heat will be for water,

$$w\frac{dt}{dx}=\pi r^2g\frac{\sigma^{2-n}}{(2r)^{3-n}}v^{n-1}\frac{\mathrm{B}^n}{\mathrm{A}}(\theta-t)$$

or, writing $w=\pi r^2\rho v$, the slope of temperature along the pipe is given by,

$$\frac{dt}{dx}=\frac{\mathrm{B}^n}{\mathrm{A}}\frac{g}{\rho}\frac{\sigma^{2-n}}{(2r)^{3-n}}v^{n-2}(\theta-t).\ .\ .\ .\ .\ .\ (6)$$

---

[*] Where the viscosity of water at 0° C. is taken at unity.

This is on the supposition that the conductivity of the water, as compared with the viscosity, does not enter; but as it probably does, for ultimately it is by conduction that the heat passes from the walls of the pipe to the water, there will probably be a coefficient $\frac{f(c)}{\sigma}$, the form of which can be determined by experiment.

If θ is practically constant, integration of the equation 6 gives, with t_1 and t_2 as the temperatures at inlet and outlet,

$$\log_e \frac{\theta - t_1}{\theta - t_2} = \frac{B^n}{A} \frac{g}{\rho} \frac{\sigma^{2-n}}{(2r)^{3-n}} v^{n-2}\, l \quad . \; . \; . \; . \; . \; . \quad (7)$$

Following on Professor Reynolds' work, Dr. T. E. Stanton* made some experiments within a limited range on the flow of heat between water and a tube surface, and the results confirmed the Reynolds law within the range of the experiments. The apparatus consisted of two concentric tubes, placed vertically, through which water was allowed to flow, a cold stream flowing through the inner tube and a hot stream through the annular space between the tubes and in the same direction. The colder stream received heat from the hot stream and this arrangement ensured that the temperature of the inner tube was nearly constant along its length. This inner tube was of drawn copper, and its mean temperature was estimated from its change of length. By altering one variable at a time Dr. Stanton deduced the following expression, applicable when $t_2 - t_1$ is small.

$$Kl = \frac{(2r)^{3-n}\, v^{2-n} \log_e \dfrac{\theta - t_1}{\theta - t_2}}{\sigma^{2-n}\,(1 + a\theta)\,(1 + \beta t_m)} \quad . \; . \; . \; . \; . \; . \; . \quad (8)$$

The terms $(1 + a\theta)$ and $(1 + \beta t_m)$ were introduced to allow for the effect of the variable conductivity of the surface film of water. If dimensions are in centimetres and temperatures in deg. Cent., it was found that $a = \cdot 004$ and $\beta = \cdot 01$, and that K had the values in Table 12.

---

* "Passage of Heat between Metal Surfaces and Liquids in contact with them," *Trans. Roy. Soc.*, 1897, Section A.

TABLE 12

Diameter of tube cms.	Length cms.	Values of K.			Number of Experiments.	Remarks.
		Max.	Min.	Mean.		
1·39	47·0	·0108	·0104	·0106	22	Flow of heat from metal to water.
1·07	44·5	·0104	·0100	·0102	13	
·736	46·0	·0103	·0099	·0100	15	

Some values of K were also deduced when the heat-flow was from the water to the metal and were found to be distinctly lower than for the flow of heat in the other direction, the average being about ·00755.

A more general explanation of the Reynolds law of heat transmission has been given by Dr. Stanton.* Speaking of the flow of heat from a surface to a fluid he remarks that "according to the Reynolds theory, if the effect of the conductivity compared with that of the viscosity be neglected, between any two cross-sections distance *dx* apart the ratio of the momentum lost by skin friction to the total momentum of the fluid is the same as the ratio of the heat actually supplied (by convection only) to the fluid between these sections and the heat which would have been supplied if the whole of the fluid flowing through the pipe had been carried up to the surface between the two sections."

Using the symbols enumerated on p. 116, this is expressed mathematically as follows :—

$$\frac{\text{Momentum lost in length } \delta x}{\text{Total momentum in fluid}} = \frac{\pi r^2 \delta p}{wv}$$

$$\text{and, } \frac{\text{Heat exchange in length } \delta x}{\left\{\begin{array}{l}\text{Total heat given if all the} \\ \text{fluid reached the surface} \\ \text{temperature in length } \delta x\end{array}\right.} = \frac{w\delta T}{w(T-\theta)}.$$

$$\text{Then, } \quad \frac{\pi r^2 \delta p}{wv} = \frac{w\delta T}{w(T-\theta)} \quad \ldots\ldots \quad (9)$$

---

* "Note on the Relation between Skin Friction and Surface Cooling," *Technical Report of the Advisory Committee for Aeronautics,* 1912–13.

If S_p is the specific heat of the fluid at constant pressure,

Heat lost per unit area of pipe per sec. $=H=\dfrac{S_p w \delta T}{2\pi r\delta x}$ . . (10)

The resistance per unit area of pipe $=R$,

and $$R=\frac{\pi r^2\delta p}{2\pi r\delta x}.$$

From equation 9,

$$\pi r^2\delta p=wv\frac{\delta T}{T-\theta}.$$

Thus, $$2\pi r\delta x=wv\frac{\delta T}{T-\theta}\frac{1}{R}.$$

Inserting this in 10,

$$H=\frac{S_p w\delta T}{wv\dfrac{\delta T}{T-\theta}\dfrac{1}{R}}=\frac{S_p R\,(T-\theta)}{v}.$$

Inserting the expression 11, p. 70, for R,

Then, $$\frac{H}{T-\theta}=h=a_1 S_p\,v\rho\left(\frac{vd}{\frac{\mu}{\rho}}\right)^{n-2} \quad . . . . . (11)$$

For a gas, $$S_v=c\,\frac{k}{\mu}\text{ (see p. 64)},$$

where k is the conductivity of the gas and c a constant varying from ·5 to ·59.

But $$S_v=\frac{S_p}{\gamma}=\frac{S_p}{1.41}\text{ for a diatomic gas.}$$

$$\therefore \quad h=a_2 k\left(\frac{v}{\frac{\mu}{\rho}}\right)^{n-1} d^{n-2} \quad . . . . (12)$$

where $a_2=\cdot78a_1$ on the average.

For a smooth tube probably $n=1\cdot75$ nearly, which gives

$$h=a_2 k\left(\frac{v}{\frac{\mu}{\rho}}\right)^{\frac{3}{4}} d^{-\frac{1}{4}} \quad . . . . (13)$$

For a very rough tube, $n=2$,

and then, $$h=a_1 S_p v\rho=a_1 S_p\left(\frac{w}{a}\right) \quad . . . . (14)$$

The results given in Table 13 from the above-mentioned report illustrate the relations between the experimental and the theoretical values of the heat transmission.

TABLE 13

Dia. of pipe cm.	Mean velocity of flow cm. per sec.	Surface temperature of pipe °C.	Fluid temperatures		Estimated friction dynes per sq. cm.	Heat transmission Calories per sq.cm.		Remarks.
			Initial. °C.	Final. °C.		Calculated	Observed	
1·39	123·2	47·2	18·0	23·92	50·6	10·8	5·36	Stanton's experiments on water.
1·39	69·0	47·3	17·98	24·45	17·1	6·5	3·28	
4·88	1480	43·5	mean of 23·5		measured 8·15	·0266	·030	Pannell's experiments on air.

There is apparently a large difference between the calculated and the observed values of heat transmission for water, and it may be assumed therefore that the conductivity effect at the boundary is not negligible for a fluid like water. For air the calculated and observed values are fairly close and it would therefore appear that for a gas the conductivity effect under the conditions of the experiment could be neglected.

Some writers have assumed that the resistance to the flow of heat between a flowing fluid and a surface is entirely due to the resistance of a film of nearly stationary fluid near the surface, and they have attempted to calculate the thickness of this film. In this connection it is interesting to note that the same expression (12), p. 119, with a small change of constant,* can be deduced in the following manner on the assumption that the resistance to heat flow is entirely due to a film of the fluid, at any rate for a gas, and, accepting the Reynolds law as true, it would follow that the presence of an actual film is somewhat doubtful.

Supposing the film to be of definite thickness x and the velocity of flow past the film to be v,* the gradient of velocity at the surface of the tube would be $\frac{v}{x}$*, and the resistance to the flow of the fluid per unit of surface (R) would be—

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* A reference to the distribution of axial velocity across the diameter of a tube shown by Figs. 19 to 22, shows that this velocity v must be much below the mean velocity of flow through the tube. If this expression is used with the mean velocity of flow inserted the result is valueless and based upon a misconception of the distribution of the axial velocity of flow.

$$R=\mu\frac{v}{x}.$$

From equation 11, p. 70,

$$R=a_1\rho v^2\left(\frac{vd}{\frac{\mu}{\rho}}\right)^{n-2}$$

$$\therefore\ \frac{1}{x}=a_1\left(\frac{v\rho}{\mu}\right)^{n-1}d^{n-2},$$

If H=heat transmitted per unit surface when the difference of temperature on the two sides of the film is $(T-\theta)$, and, k=conductivity of the fluid.

Then, $\frac{H}{T-\theta}=h=\frac{k}{x}$,

$$\text{or, } h=a_1k\left(\frac{v\rho}{\mu}\right)^{n-1}d^{n-2} \quad . \ . \ . \ . \ . \ . \ . \ . \quad (15)$$

For a liquid, however, the equations cannot be so definitely related.

The above discussions relate principally to the flow of the fluid parallel to the surface. Some special experiments have been made by Jakemann† on the cooling effect with the section across a stream of air flowing at 30 ft. per sec.

Referring to the sketch, Fig. 40, the cooling or heating effect at A, B, and C may be widely different. The motion at A will probably be slower than at B, and C is acted on only by the eddies thrown off the sides.

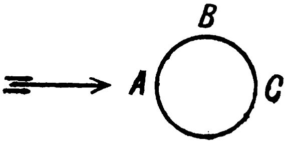

FIG. 40.—Flow of fluids across tubes.

To test the different cooling effects on these three regions, a thin strip of platinum foil, 180 mm. long, 2·5 mm. wide, and about ·01 mm. thick, was fixed on the surface of an ebonite rod 20 mm. diameter, with its length parallel to the axis. A current was passed through this, and the heat dissipated was estimated by Mr. Jakeman for given ranges of temperature between the surface strip and the air. It is seen from Fig. 41 that for a given temperature difference the heat dissipated is greatest for the surface immediately facing the current. This was unexpected* and the strips were taken off and attached to the centre of the face of a prism of square section 40 mm. by 40 mm., and the results obtained are shown in Fig. 42. Here it is seen that the cooling effect is a maximum in the eddying region at the back, and least for the normal side facing the current. Comparing the results with those for the round rod it is seen that the cooling effect at the back is practically the same for both cases, whereas for the normal

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* See footnote, p. 120.

† See p. 118 for reference.

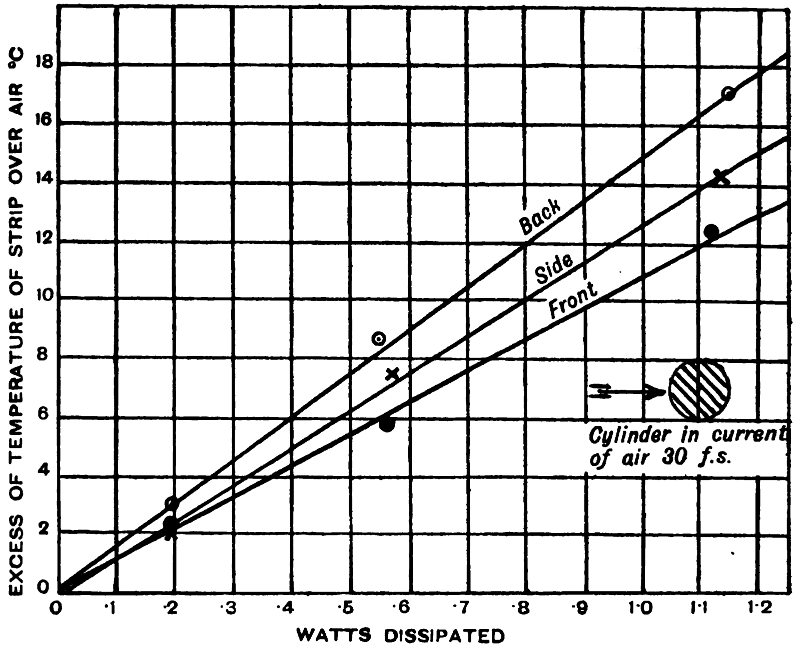

Fig. 41.—Dissipation of heat from a circular surface in a cross-current of air.

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* One would naturally anticipate that the flow of air round the rod would leave more or less of a dead space opposite A, but possibly this would depend somewhat on the velocity of the air.

position facing the current, the loss from the cylinder was more than twice that for the prism.

These results are, of course, purely relative, as conduction into the prism is also included in the heat loses ; but Dr. Stanton states " there is reason to believe that even when conduction losses are eliminated, the heat dissipated under these conditions is considerably greater than under the conditions of simple skin friction obtained in a pipe experiment."

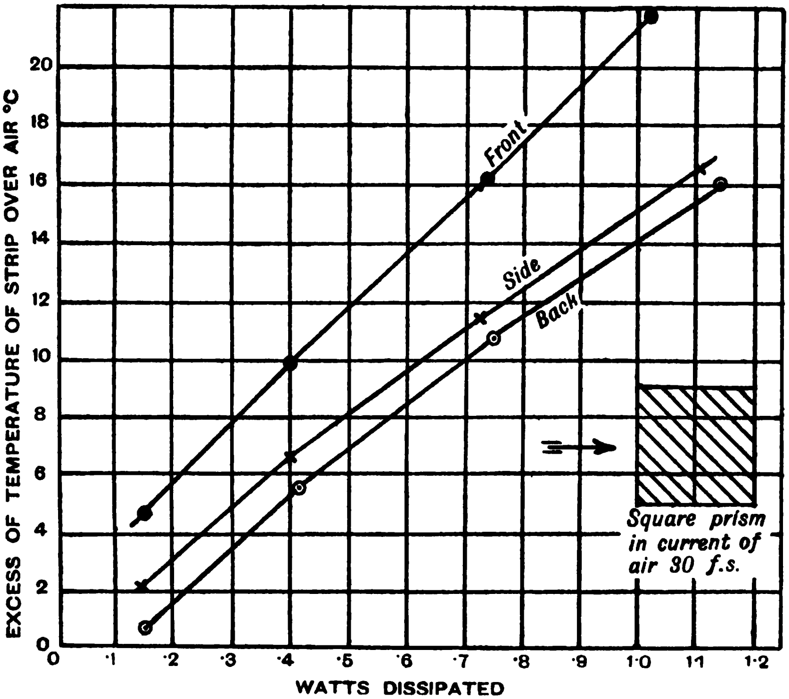

FIG. 42.—Dissipation of heat from a square-shaped surface in a cross-current of air.

An elaborate and interesting series of experiments on the rate of heat transmission between a tube and various gases at temperatures a little above that of the atmosphere have been made by W. Nusselt.* The apparatus used is illustrated in

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* "Der Wärmeübergang in Rohrleitungen," *Zeitschrift des Vereines deutscher Ingenieure,* 23rd and 30th October, 1909.

Fig. 43, consisting of a brass tube C 26 mm. (1·02 in.) outside and 22 mm. (·866 in.) inside diameter, surrounded by the tube D. Steam was supplied through a cock so as to keep the tube at a temperature of about 102° C., this tube temperature being measured by the thermo-junction *b*. The experiments were made with air, coal gas, and carbon-dioxide, which gases were allowed to flow into the tube at A and to leave at B through the cock E, the volume being then measured by special meters. The average temperature of the gases when flowing through the tube would be about 35° C., but some difficulties were experienced in measuring the true temperature of the gas. Special experiments were conducted on a special cylinder to estimate the influence of radiation and convection on the measurement of temperature and it was found that the best estimation of temperature was obtained by means of a platinum electrical resistance thermometer of thin wire stretched across the cylinder at right angles to the current of gas. In the apparatus shown in Fig. 43, under some conditions of flow as much as 5° C. (9° F.) difference of temperature of the gas was found in the cross-section of the 22 mm. tube. The arrangement finally adopted to measure the gas temperature was a platinum resistance thermometer made of ·1 mm. (·0039 in.) diameter wire, wound in four turns over a mica cross, as shown in the cross-section *a*, Fig. 43, and mounted on ebonite attached to a brass tube 5 mm. diameter. This passed axially through a stuffing-box at the end of the pipe or tube, so that it

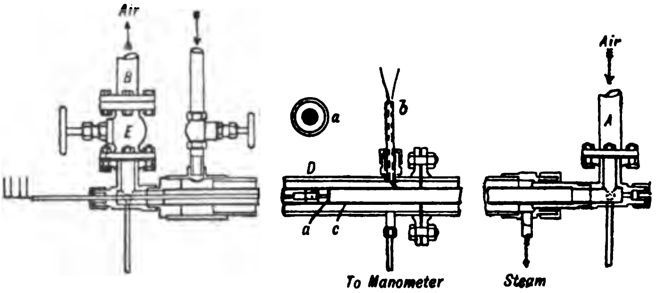

FIG. 43.—Nusselt's apparatus for measuring the transmission of heat to gases flowing through a tube.

could be placed in any convenient position along the length of the tube. The first temperature measurement of the gas was usually made at 15 cm. (5·9 in.) from the elbow and the second point at a half-metre or a metre further on. Sometimes another pipe 2 metres in length was inserted between the inlet elbow and the point of measurement of the temperature to quiet the eddies at velocities below the critical velocity.

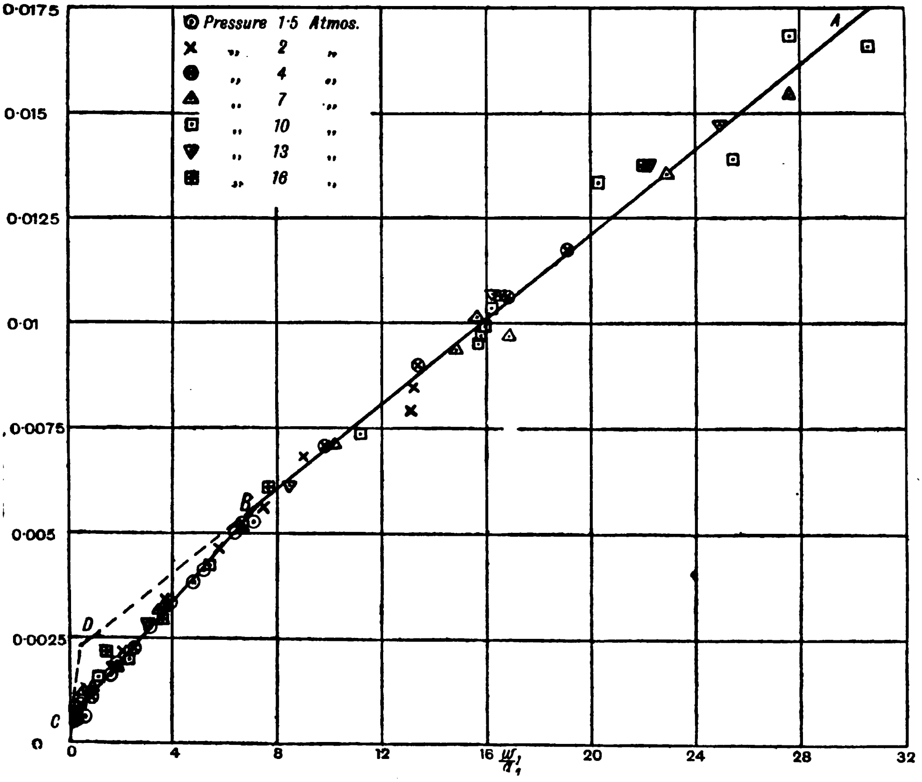

FIG. 44.—Rate of heat transmission to air flowing in a tube at various pressures, from Nusselt's experiments.

When air was used it was obtained from the receiver of a compressor and allowed to flow through the tube at pressures ranging from 1·15 atmospheres to 16 atmospheres and at various velocities. With coal gas and with carbon-dioxide the experiments were made only at one definite pressure.

In the original paper Nusselt plotted the rate of heat transmission on a velocity base, but did not tabulate the gas temperatures. The writer has converted the rates of heat transmission given by the values in the graphs into B.Th.U. per second per square foot per deg. Fahr. average difference between the tube and the gas and converted the rate of gas flow into terms of $\frac{\text{pounds per second}}{\text{area of flow, sq. ft.}}=\frac{w_1}{a_1}$. The results obtained for air are shown graphically in Fig. 44 and it is seen that the pressure of the air had little or no effect on the rate of heat transmission expressed in terms of $\frac{w_1}{a_1}$, as should be the case if the Reynolds law holds. Although a smooth mean curve might be sketched in among the points to represent average values, it would be noticed that, to all intents and purposes, the average values could equally well be represented by the two straight lines AB and BC.

From the results of his experiments Nusselt deduced the following equations to represent the rate of heat transmission in the tube used.

$$h=b(v_1\rho_1)^{\cdot 786} \text{ or } b\left(\frac{w_1}{a_1}\right)^{\cdot 786} \text{ at about } 35^\circ \text{ C.}$$

where b had the following values :—

If h is in kgm., hour, square metre, °C., units,

v_1 in metres per second,

ρ_1 in kgms. per cubic metre,

w_1 in kgms. per second,

a_1 in square metres,

then $b=5{\cdot}772$ for air, 4·774 for carbon-dioxide, 15·45 for coal gas, and, deduced from Knoblauch and Jakob's experiments on the specific heat of superheated steam, 10·79 for superheated steam.

If h is in B.Th.U. per sq. ft. per sec. per deg. Fahr. temperature difference,

v_1 in ft. per sec., ρ_1 in lbs. per cubic ft.,

w_1 in lbs. per sec., a_1 in sq. ft., then $b = \cdot 00114$ for air, ·000942 for carbon-dioxide, ·00306 for coal gas, and deduced for superheated steam ·00213.

Referring to Fig. 44, however, it would be seen that the rate of heat transmission for air could be expressed by two straight lines in these experiments of the form,

$$h = \text{A} + \text{B}\rho_1 v_1 = \text{A} + \text{B}\frac{w_1}{a_1}.$$

Probably with a rougher tube the resistance to the flow of the fluid through the tube would have been more nearly proportional to v^2, and in that case the straight line AB would have been somewhat higher on the graph and probably would have continued its direction nearly to the zero line of velocity in the manner indicated by the dotted line BDC, the sudden drop or discontinuity at D signifying that the velocity of flow was below the critical value.

If the Reynolds law stated on p. 119 is substantially true the values of b for the different gases experimented upon by Nusselt should be roughly proportional to the specific heat at constant pressure S_p. That this is found to be the case is seen from the values in Table 14.

TABLE 14

Gas.	Value of b.	Approximate value of S_p.	Ratio $\frac{b}{\text{S}_p}$.
Air	·00114	·24	·00475
Carbon-dioxide	·000942	·2	·00471
Coal gas	·00306	·6	·0051
Superheated steam	·00213	·48	·00445

Experiments have been made by H. P. Jordan, working under the late Professor Nicolson at the Manchester College of Technology,* with the object of proving, within the limits of the experiments, the Reynolds law of heat transmission for fluids like air and water. The apparatus consisted of the air cooler or calorimeter represented in Fig. 45 together with the necessary measuring arrangements. Air was supplied from an air-compressor to a suitable receiver from which the air was passed through a heating coil immersed in the salt bath of a Brayshaw hardening furnace. The air then passed through the air cooler or calorimeter and was finally discharged through a sharp-edged orifice in an air discharge box.

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* *Inst. of Mech. Engs.*, 1909, p. 1317, "On the Rate of Heat Transmission between Fluids and Metal Surfaces."

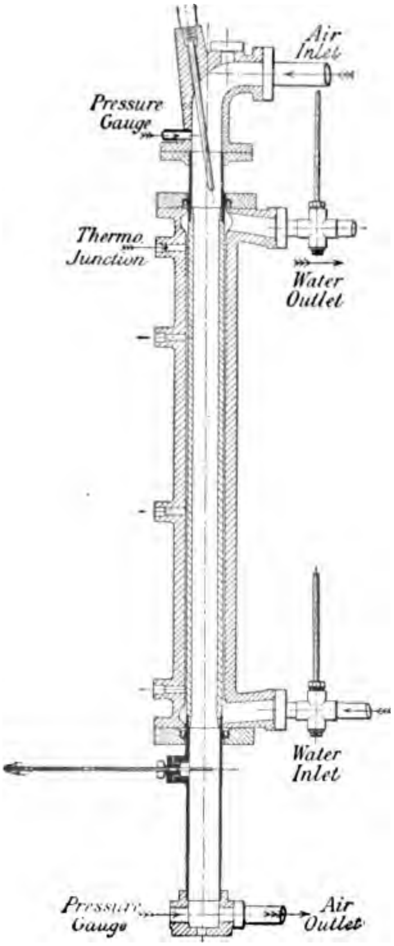


FIG. 45.—Heat transmission apparatus used by Jordan.

Referring now more particularly to the air cooler shown in Fig. 45, the air was passed through the copper pipe and its temperature was measured at the inlet by means of a platinum resistance thermometer and Whipple indicator, and at the outlet by a mercury thermometer. The copper pipe was surrounded by a cast-iron casing and the space between formed a water jacket. A heating coil was also provided so that the water could be supplied at any temperature up to 200° F. As shown the counter-current principle of flow was adopted and the inlet and outlet temperatures of the water were measured by mercury thermometers, whilst the quantity of water flowing was determined by weighing.

The greatest care was taken in the design to eliminate all possible sources of error due to heat conduction. The copper pipe was screwed to steel ends which were made very thin to avoid end effects due to conduction of heat along the pipe into and out of the apparatus. Also, contact between the pipe and the casing was avoided by making the joints at each end by means of cup leathers.

In order to test what effect, if any, change of dimensions or section of channel would have on the rate of heat transmission, five series of experiments were made with different air passages, two of which were made with annular air passages and three with open pipes. The changes of air passage for the open pipe series were made by unscrewing the copper pipe from the steel studs and fitting to the same ends a pipe of different diameter, the casing being bushed up to whatever extent was necessary to give the required water space. The experiments with annular pipes were made with the largest copper pipe partially stopped by a wrought-iron core held centrally in the pipe by means of a wire spiral, two cores of different diameters being used in this series.

The temperature of the copper tube was measured at the four places indicated in Fig. 45 by means of constantan-copper thermo-junctions. The detail construction and arrangements of these thermo-junctions is given on p. 120 of *The Measurement of Steady and Fluctuating Temperatures*, so that further description of these is unnecessary.

The five series of experiments were distinguished by the letters B, C, D, E, and F, each series having a different area of air passage. Table 15 gives information as to the form and dimensions of the various air and water spaces. The symbols m_1 and m_2 refer to the following ratio for the air and water passages respectively, $\frac{\text{area of flow, sq. in.}}{\text{perimeter, inches}}$.

TABLE 15

Air Passage.					Water Passage (Annular).		
Series.	Section for air-flow.	Area in sq. ft.	m_1 inches.	Area of cooling surface sq. ft.	Area in sq. ft.	m_2 inches.	Area of heating surface sq. ft.
B .	Annular .	·00725	·171	1·69	·0059	·0977	2·38
C .	,, .	·01103	·26	1·69	·00311	·0972	2·38
D .	Circular .	·0211	·492	1·69	·00311	·0972	2·38
E .	,, .	·001397	·1265	·435	·00264	·143	·857
F .	,, .	·00834	·309	1·062	·003535	·1302	1·51

Counter current flow was adopted throughout and the following list of symbols were used in the calculations :—

T=air temperature.
T_1 and T_2=inlet and outlet temperatures of the air.
t=water temperature.
t_2 and t_1=inlet and outlet temperatures of the water.
θ=temperature of metal surface.
θ_i=mean temperature of the inner surface of the pipe (gas side).
θ_T and θ_B=temperatures of the air cooling surface at the top and bottom of the calorimeter respectively.
l=length of cooling surface of calorimeter in feet.
a_1=area of air passage through calorimeter, sq. ft.
a_2=area of water passage through calorimeter, sq. ft.
w_1=lb. of air passing per second.
w_2=lb. of water passing per second.
H=B.Th.U. gained by water per second.
h_1=B.Th.U. from the air into the metal per square foot, per second, per deg. Fahr. difference between air and metal.
h_2=B.Th.U. from the metal to the water to the same units.
d_1=internal diameter of pipe.
d_2=external diameter of pipe.

The weight of air-flow w_1 was calculated from the heat given to the cooling water, assuming external heat losses to be negligible, thus,

$$H=w_2(t_1-t_2)=\cdot 238\ w_1(T_1-T_2)$$

$$\text{or, } w_1=\frac{w_2(t_1-t_2)}{\cdot 238(T_1-T_2)} \quad \ldots\ldots\ldots\ldots \quad (16)$$

This weight of air-flow was checked by calculations based on the pressure and temperature of the air in the air-discharge box measured near the orifice.

The measured temperatures of the tube were obtained at a depth of half the thickness of the tube, and the corresponding temperatures at the outer and inner surfaces were calculated by means of an expression similar to equation 3 on p. 24.

Plotting the measured metal temperatures on a length base gave curves similar to A–B in Fig. 46 which were nearly straight lines, and in working out the results, for the sake of simplicity, they were taken as straight lines. The mean temperature of the cooling surface is therefore θ_i in Fig. 46, and the straight line joining A–B gives the average slope of the temperature gradient down the cooling surface. A line, therefore, parallel to the dotted line AB through a temperature θ_i at a point half-way down the tube will have about the same average slope as the true curve, and will give the same mean temperature of the cooling surface.

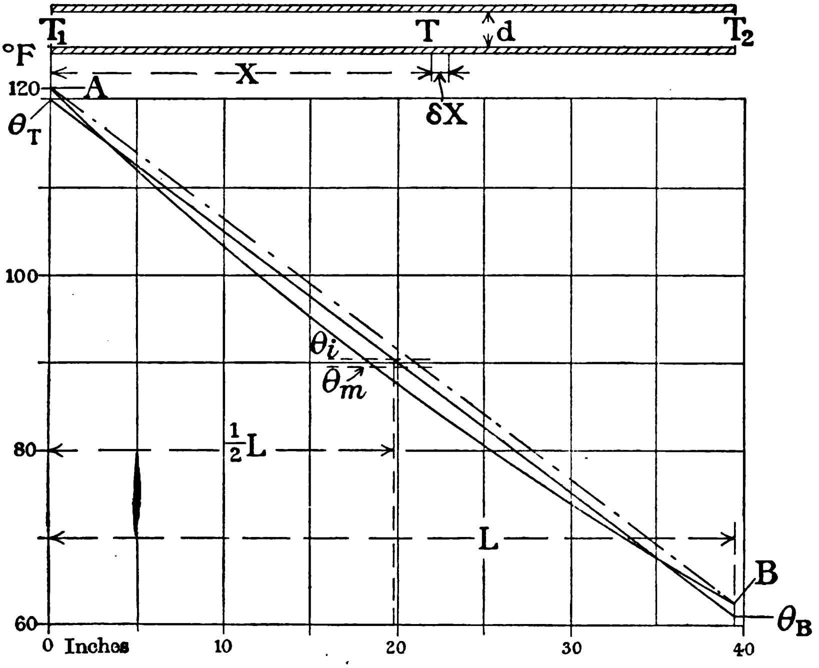


FIG. 46.—Temperature gradient along length of tube.

In the calculations such a line has been taken to represent the temperature gradient along the length of the tube.

Consider now any element of heating surface, $\pi d_1 \delta x$, distant x from the top of the tube.

Then, $-w_1 s\delta T = h_1(T-\theta)\pi d_1\delta x$.

Now, $\dfrac{d\theta}{dx} = \dfrac{\theta_T - \theta_B}{l} = c$.

Therefore, $\theta = \theta_T - cx$.

or, $-sw_1\delta T = h_1\{T-(\theta_T - cx)\}\pi d_1\delta x$,

or, $-\dfrac{dT}{dx} = \dfrac{h_1\pi d_1}{sw_1}\{T-(\theta_T - cx)\}$,

or, $\dfrac{dT}{dx} + \dfrac{h_1\pi d_1}{sw_1}T = \dfrac{h_1\pi d_1}{sw_1}(\theta_T - cx)$

putting $\dfrac{h_1\pi d_1}{sw_1} = a$, this becomes,

$$\frac{dT}{dx} + aT = a(\theta_T - cx).$$

The solution to this differential equation is,

$$al = \log_e \frac{T_1 - \theta_T - \dfrac{c}{a}}{T_2 - \theta_B - \dfrac{c}{a}} \quad \ldots\ldots\ldots\ldots \quad (17)$$

All the values in this equation would be known from the experimental results except a, and thus solving for $a = \dfrac{h_1\pi d_1}{sw_1}$, the value of h_1 is calculable.

The mean air temperature can now be obtained by dividing the heat transmitted per square foot per second by the coefficient h_1 (which first gives the mean difference of temperature).

Thus, $$T_m - \theta_i = \frac{H}{A_1 h_1}$$

and since θ_i is known, the mean air temperature T_m may be calculated.

The comparisons in Table 16 have been made from values selected at random from Mr. Jordan's paper, in order to illustrate the errors which would be involved by taking the mean air temperature to be the arithmetical mean temperature $\dfrac{T_1+T_2}{2}$ instead of the more correct value of T_m deduced by the method just described.

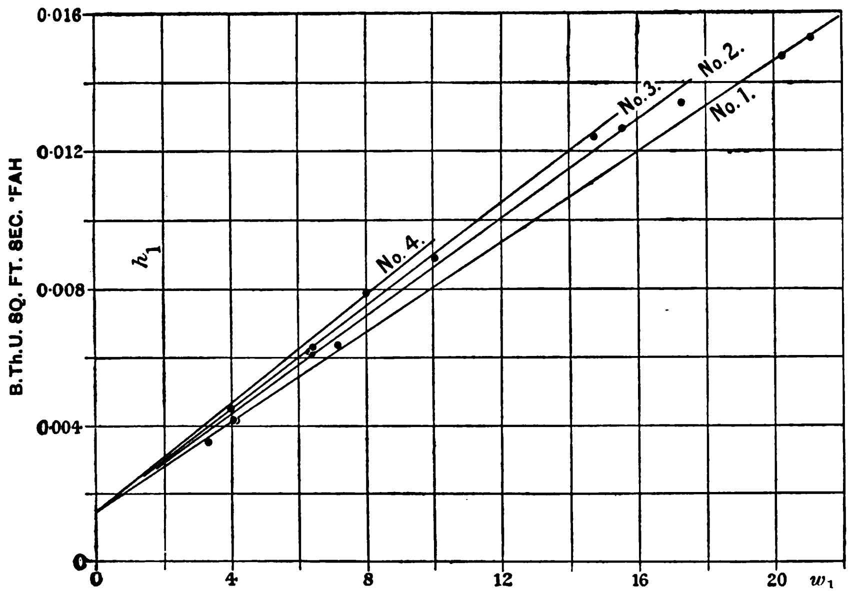


FIG. 47.—Rate of heat transmission from gas to surface in Jordan's experiments. Series B.

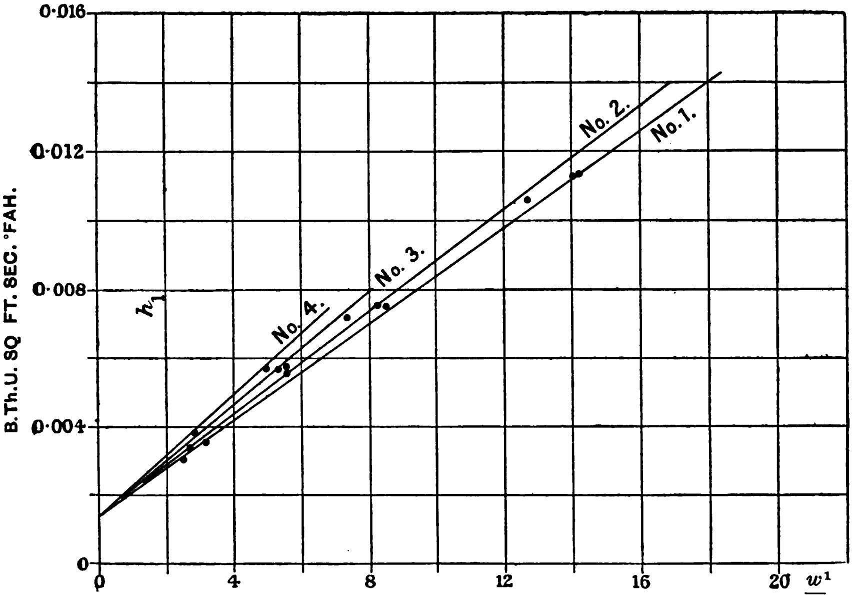


FIG. 48.—Rate of heat transmission. Series C.

TABLE 16

Inside diameter of tube d_1 inches	Air Temperatures.					Water Temperatures.		$T_m-\theta_i$ °F.
	T_1 °F.	T_2 °F.	$\frac{T_1+T_2}{2}$ °F.	T_m °F.	$\frac{T_1+T_2}{2}-T_m$ °F.	Inlet. t_2 °F.	Outlet. t_1 °F.	
1·968	340	235	287·5	283·7	3·8	50·74	54·44	228
	404	308	356·0	352·8	3·2	53·9	62·5	288
	582	403	492·5	478·7	12·8	59·0	75·1	402
1·236	415	260	337·5	331·1	6·4	72·4	82·6	246
	607	383	495	483·4	11·6	79·0	86·0	392
	736	443	589·5	583·1	6·4	79·5	88·4	490
0·506	372	154	263·0	242·3	20·7	61·0	72·5	165
	440	177	308·5	295·3	13·2	63·9	80·6	209
	637	245	441	402·7	38·3	82·0	99·0	294

It would be noted that the difference $\frac{T_1+T_2}{2}-T_m$ is generally greater the greater the fall of air temperature (T_1-T_2) through the tube. The quantity $\frac{T_m+\theta_i}{2}$ would give a rough measure of the mean temperature of any supposed film of gas adhering to the cooling surface.

The results of the experiments are shown graphically in Figs. 47 to 51, where h_1 is shown plotted on a base of rate of air-flow, expressed in pounds per second per square foot of area of channel $\frac{w_1}{a_1}$. It will be seen that there is a linear law connecting h_1 and $\frac{w_1}{a_1}$ and each separate line in the figures is a mean line among the points where the value of $\frac{T_m+\theta_i}{2}$ was approximately the same. These results confirm Reynolds law within the limits of the experiments, that is,

$$h_1=\left(A+B\frac{w_1}{a_1}\right) \text{ or } (A+Bv_1\rho_1) \quad . \; . \; . \; . \; . \; . \; . \; . \quad (18)$$

Fig. 52 shows the values of the factor B obtained from Figs. 47 to 51 plotted on a base of supposed film temperature $\frac{T_m+\theta_i}{2}$, and with the exception of series F, the points lie on

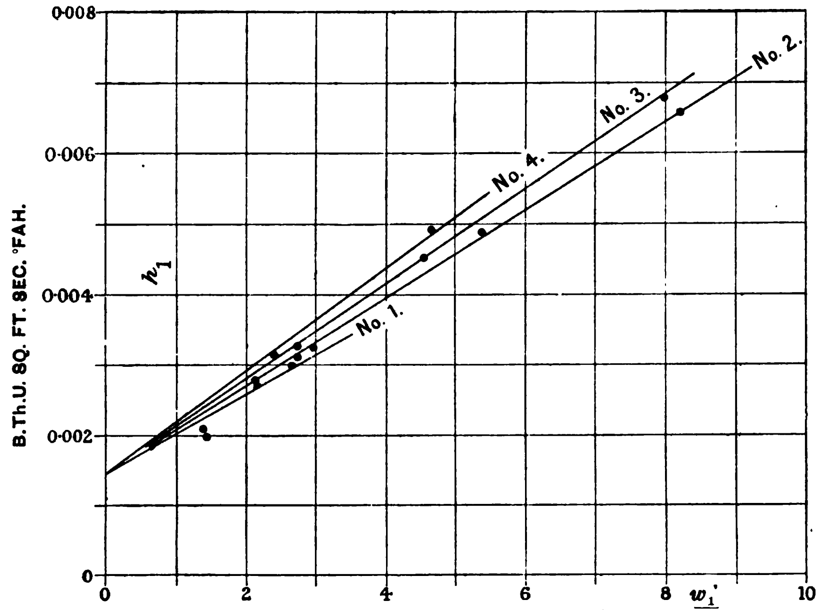


FIG. 49.—Rate of heat transmission. Series D.

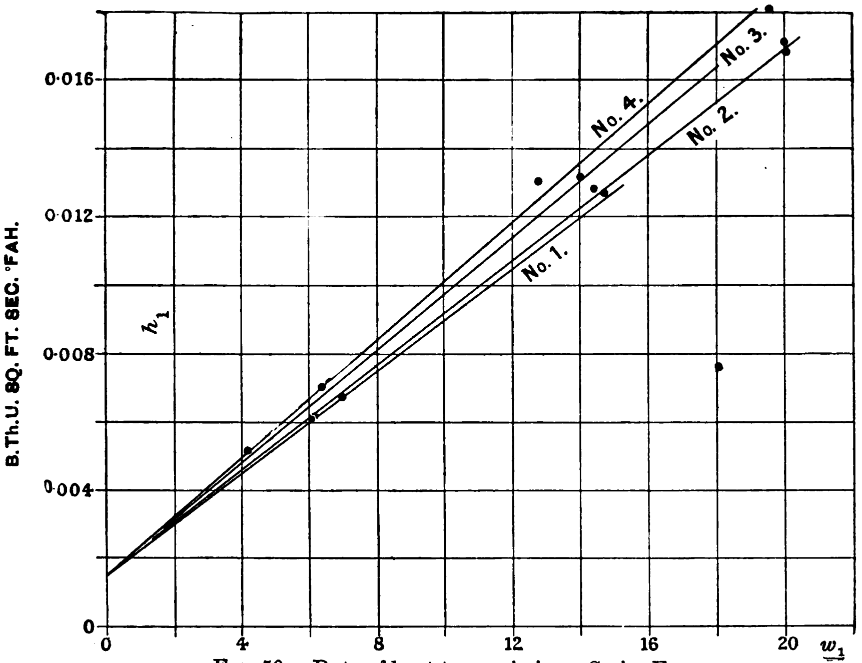


FIG. 50.—Rate of heat transmission. Series E.

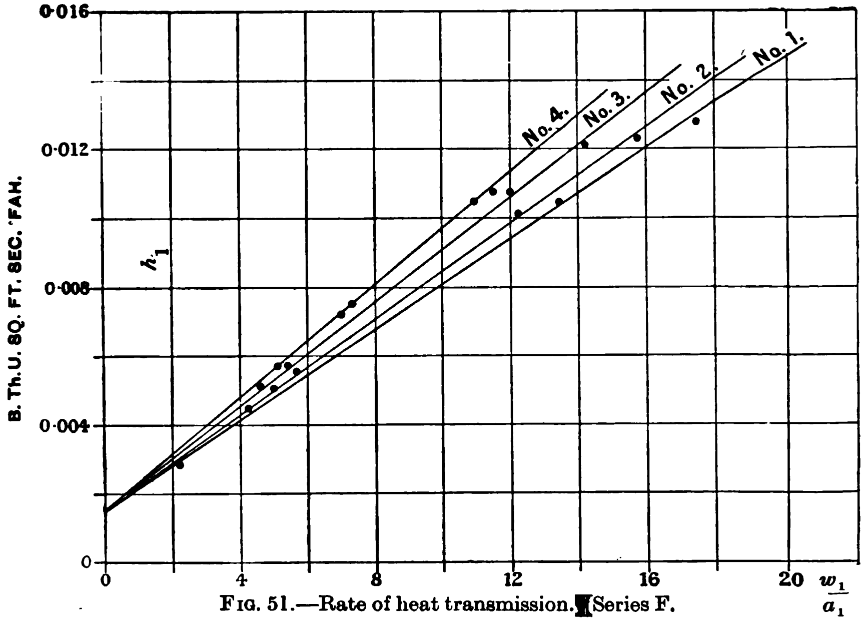


FIG. 51.—Rate of heat transmission. Series F.

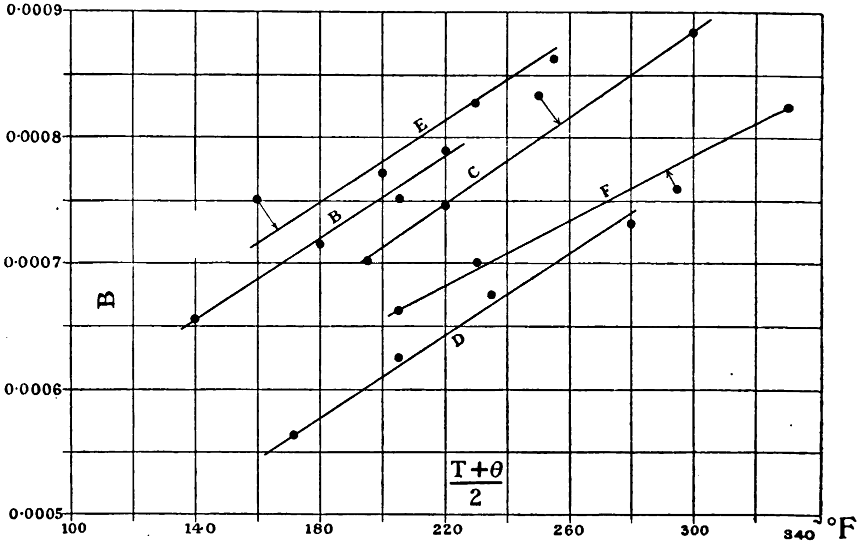


FIG. 52.—Variation of "B" in equation 18, from Jordan's experiments.

a number of parallel lines, thus giving $B = X + Y\frac{T_m + \theta_i}{2}$, where X is a function of the dimensions of the channel. The values of X plotted against the hydraulic mean depth m_1 gave a straight line, the equation being :—

$$X = \cdot 000506 - \cdot 00045 m_1$$

The average value of Y was found to be ·00000165. Hence, to sum up, the values of A and B in the above equation are— A = ·0015 and B, which is a function of the dimensions and the temperatures, is given by

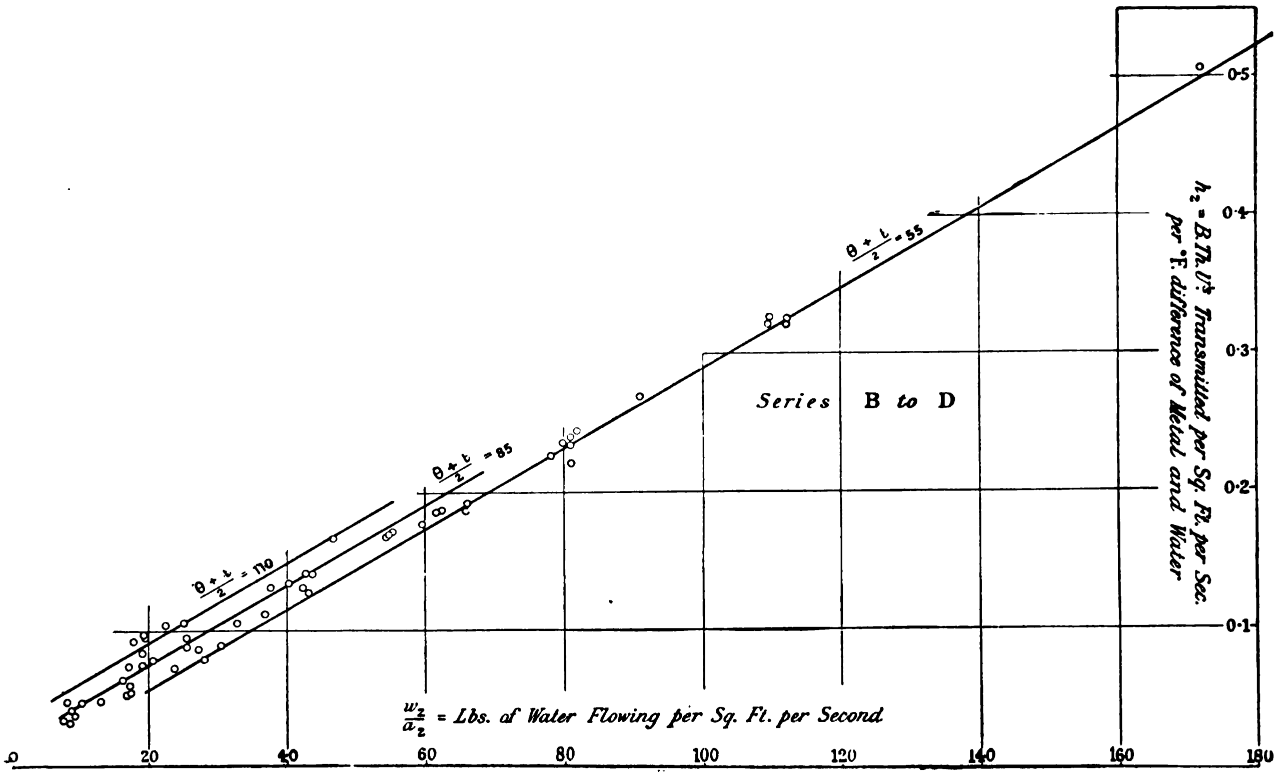
$$B = \cdot 000506 - \cdot 00045 m_1 + \cdot 00000165 \left(\frac{T_m + \theta_i}{2}\right).$$

It will be noted then that the smaller the value of m_1 and the greater the value of $\frac{T_m + \theta_i}{2}$ the greater is the value of B, and the rate of heat transmission is accordingly greater. The general law expressed by equation 11, p. 119, suggests the probability of a small variation of heat transmission with d_1 (i.e. with m_1) when the index n is less than 2, which is probably the case in these experiments, and therefore the variation of B with m_1 in these experiments in some measure confirms this law.

The A term in the Reynolds law represents the effect of conduction and natural diffusion in a stationary gas. It would therefore be expected that the value of A would increase to some extent with the temperature of the gas, but Jordan's experiments do not indicate this. A reference to the plotted results in Figs. 47 to 51, however, suggests that the points available are not sufficiently numerous to determine any variation of A with certainty, and therefore the results do not constitute a proof that some variation does not exist.

Mr. Jordan did not record any of the air pressures in his paper and presumably the experiments were all made at pressures which were nearly atmospheric, and therefore the only appreciable variations in the density were due to changes of temperature.

The experimental values of h_2 are plotted in Figs. 53 and 54 on a base of $\frac{w_2}{a_2}$. It is seen that in each series there appears to



be a definite straight line law within the limits of the experiments and that the value of h_2 depends slightly upon the mean temperature $\frac{\theta_0+t}{2}$, where t was taken to be $\frac{t_1+t_2}{2}$ Qualitatively they confirm Stanton's results, but refer in all cases to annular flow. It will be found, however, that other experiments made on steam condensers with much higher values of $\frac{w_2}{a_2}$ show that the results in Figs. 53 and 54 cannot be accepted outside the range of the experiments.

In series B and D a fine screw thread was cut on the outer surface of the tube with the idea of breaking up the flow of water. In series E and F the tube surface was plain. Comparing the results in Fig. 53 with those in Fig. 54 it is seen that the broken surface of the tube on the water side had no marked effect on the heat transmission. A consideration of the distribution of temperature from the air to the water would lead one to expect that the breaking up of the water-flow on the water side could have but little effect on the flow of heat from the air to the water under the conditions of the experiments. Thus Fig 55 has been drawn to illustrate the character of the temperature distribution in two cases taken from the series F. In test F1, the rates of flow of air and water are respectively 2·4 and 20·5 lb. per square foot per second, the total heat transmitted being ·739 B.Th.U. per second ; whereas in test F4 with the corresponding rates of flow 17·45 and 98 respectively the total heat transmitted was 3·39 B.Th.U. per second. Seeing that the temperature drop from metal to water is so small, there cannot be any large reduction of the metal temperature by increasing the velocity or disturbance of the water. Thus, the greater heat-flow in test F4 was due to the reduction of the resistance on the air side by the increased flow of air.

Fig. 55 again indicates that with high gas temperatures and with nearly clean plates, no great error would be involved in making calculations on heat transmission from a gas to water by assuming the temperature of the wall to be that of the water.

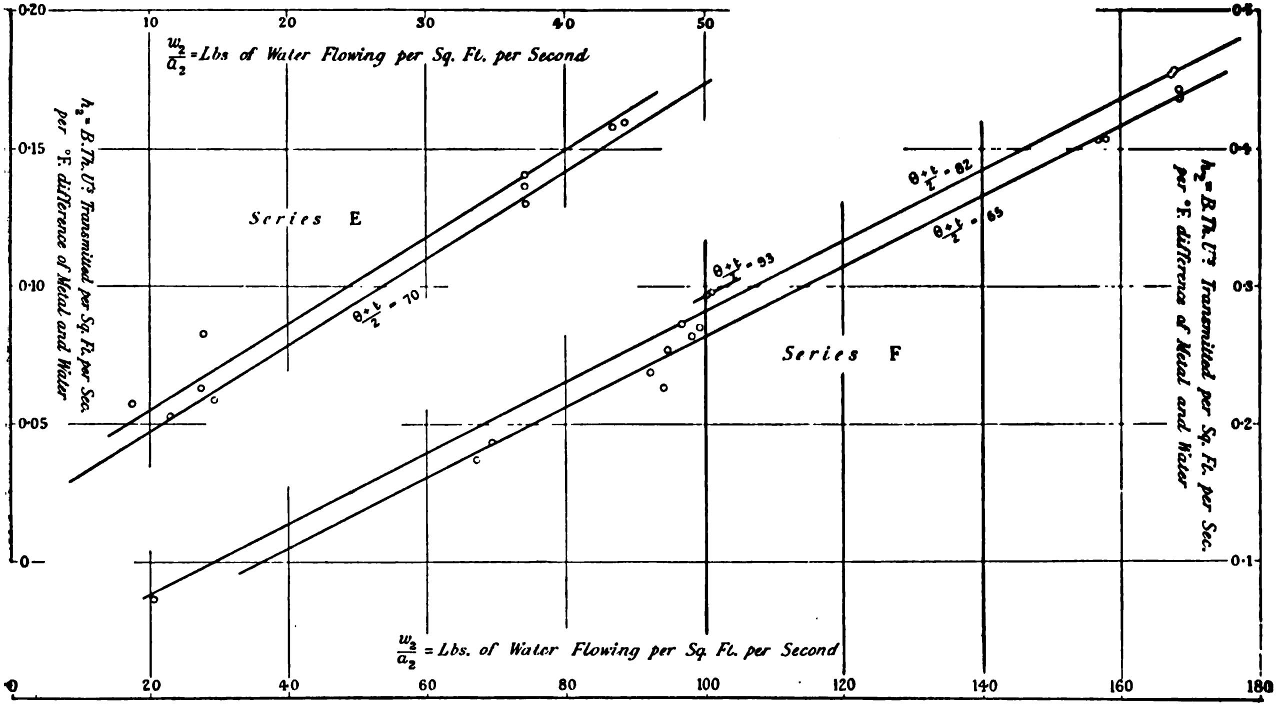


FIG. 54.—Rate of heat transmission from metal to water, from Jordan's experiments.

The results of other experiments on the flow of heat between a tube surface and water will be found on p. 195 of *Heat Transmission in Boilers, Condensers, and Evaporators.*

Further Experimental Data.—So far in this chapter we have been mostly concerned with the explanation and discussion of the fundamental basis of the Reynolds law of heat transmission by convection, and it is now proposed to consider the transmission of heat more in relation to the design of boilers and similar plant. Although innumerable tests of a more or less isolated character have been made on boilers, comparatively few complete tests have been made to ascertain what influence the variable factors have on the rate of heat transmission from the gas to the water, such as, for example, the rate of gas flow and the temperature of the gases. To anyone acquainted with the costly and uncertain nature of boiler experiments this is not surprising, and it would seem to be a more reasonable method of experimenting to employ small apparatus and to study the effect of these variable factors in the light of Reynolds law, and then to ascertain whether the results could be reasonably applied to full-sized boilers. In what follows it will be taken for granted that no sensible error is involved in assuming that, under ordinary conditions of operation with fairly clean tubes the heated surface is practically at the same temperature as the water when compared with the temperature of the gas. Experiments and apparatus will be described in some detail to give a proper idea of the relative value of the particular experiments discussed, and also to show the particular methods adopted. Before doing that, however, some general formulæ will be discussed.

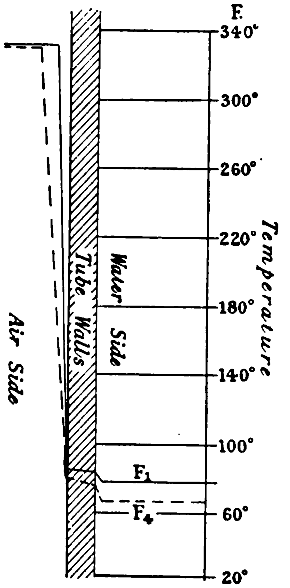


Fig. 55.—Distribution of temperature from gas to water through a tube. Jordan's experiments.

The Mean Difference of Temperature between Fluids.—A common method of stating the rate of heat transmission between fluids across a wall when the wall temperature itself is not under consideration is to express it in heat units per unit area of one of the surfaces per unit of time per degree difference of temperature between the fluids. Generally speaking the fluids alter in temperature from the inlet to the outlet (except where evaporation or condensation occurs at constant temperature) and thus it is necessary to have an estimate of the mean difference of temperature in order to be able to calculate the rate of heat transmission.

Let h=average rate of heat transmission.
T_1=inlet temperature of hot fluid.
t_1=temperature of colder fluid at same end.
T_2=outlet temperature of hot fluid.
t_2=temperature of colder fluid at same end.
t_m=mean temperature difference between fluids.
w_1=weight of hot fluid flowing per second.
w_2= ,, cold ,, ,, ,,
s_1=specific heat of hot fluid (assumed constant).
s_2= ,, ,, cold ,, ,, ,,
A=area of surface.

If the hot fluid is condensing at constant temperature obviously $T_1=T_2=T$, and if the cold fluid is evaporating then $t_1=t_2=t$. There will thus be several standard cases to consider.

1. Hot fluid condensing, colder fluid rising in temperature.
2. Hot fluid falling in temperature, cold fluid evaporating.
3. Hot and cold fluids both changing in temperature and flowing in opposite directions (counter current).
4. Hot and cold fluids both changing in temperature and flowing in the same direction (parallel current).

In the following calculations it will be assumed that the heat transmission at any point of the surface for any particular case is simply proportional to the difference of temperature between the fluids at that place. It is further assumed that there is no appreciable conduction of heat from the hot end of the surface towards the cold end and that the specific heats of the fluids are constant.

Case 1 as in Fig. 56.

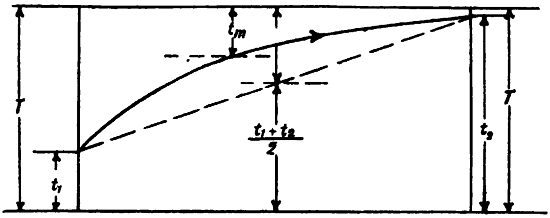


FIG. 56.—Temperatures of fluids along length of tube when hot fluid is condensing.

Let δA be an element of the surface where the hot fluid is at temperature T and the cold fluid at temperature *t*.

Then, $s_2w_2\delta t = h(\mathrm{T}-t)\,\delta\mathrm{A}$

$$\text{or, } \frac{h\mathrm{A}}{s_2w_2} = \int_{t_1}^{t_2}\frac{dt}{\mathrm{T}-t} = \log_e\frac{\mathrm{T}-t_1}{\mathrm{T}-t_2}$$

But, for the whole area,

$$ht_m\mathrm{A} = s_2w_2(t_2-t_1)$$

$$t_m = \frac{s_2w_2(t_2-t_1)}{h\mathrm{A}}$$

$$= \frac{t_2-t_1}{\log_e\frac{\mathrm{T}-t_1}{\mathrm{T}-t_2}} \text{ or } \frac{(\mathrm{T}-t_1)-(\mathrm{T}-t_2)}{\log_e\frac{\mathrm{T}-t_1}{\mathrm{T}-t_2}} \quad \ldots\ldots\ (1)$$

Case 2 as in Fig. 57.

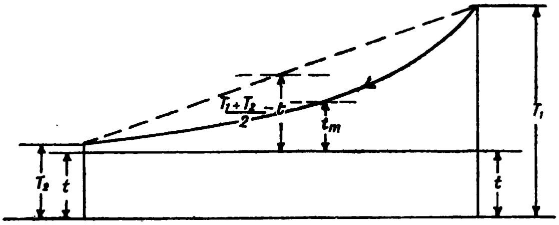


FIG. 57.—Temperatures of fluids along length of tube when colder fluid is evaporating.

By a similar calculation to that for case 1,

$$t_m = \frac{\mathrm{T}_1-\mathrm{T}_2}{\log_e\frac{\mathrm{T}_1-t}{\mathrm{T}_2-t}} \text{ or } \frac{(\mathrm{T}_1-t)-(\mathrm{T}_2-t)}{\log_e\frac{\mathrm{T}_1-t}{\mathrm{T}_2-t}} \quad \ldots\ldots\ (2)$$

Case 3 as in Figs. 58 and 60.

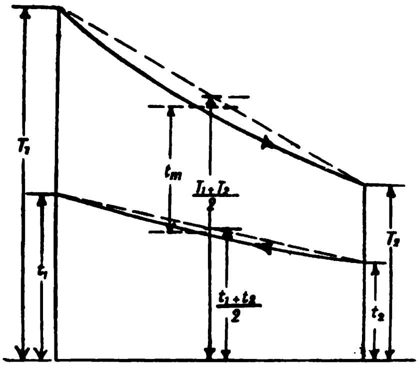


FIG. 58.—Temperatures of fluids with counter-current flow.

For the element of area δA where the temperatures of the hot and cold fluids are respectively T and t.

$$h(T-t)\delta A=-s_1w_1\delta T=-s_2w_2\delta t$$

$$\text{or, } h\delta A=-s_1w_1\frac{\delta T}{T-t}=-\frac{s_2w_2\delta t}{T-t},$$

$$\text{Thus, } hA=-s_1w_1\int_{T_1}^{T_2}\frac{dT}{T-t}=-s_2w_2\int_{t_1}^{t_2}\frac{dt}{T-t} \quad . \quad . \quad . \quad . \quad . \quad (3)$$

Between T and t, however, there is the simple relation,

$$s_1w_1(T_1-T)=s_2w_2(t_1-t)$$

$$\text{or, } T=T_1-\frac{s_2w_2}{s_1w_1}t_1+\frac{s_2w_2}{s_1w_1}t$$

$$\therefore\ T-t=T_1-\frac{s_2w_2}{s_1w_1}t_1+\left(\frac{s_2w_2}{s_1w_1}-1\right)t.$$

Inserting in equation 3,

$$hA=-s_2w_2\int_{t_1}^{t_2}\frac{dt}{T_1-\frac{s_2w_2}{s_1w_1}t_1+\left(\frac{s_2w_2}{s_1w_1}-1\right)t}$$

$$=\frac{s_2w_2}{\left(\frac{s_2w_2}{s_1w_1}-1\right)}\log_e\frac{T_1-\frac{s_2w_2}{s_1w_1}t_1+\left(\frac{s_2w_2}{s_1w_1}-1\right)t_1}{T_1-\frac{s_2w_2}{s_1w_1}t_1+\left(\frac{s_2w_2}{s_1w_1}-1\right)t_2}.$$

But since $s_2w_2(t_1-t_2)=s_1w_1(T_1-T_2)$

and, $\frac{s_2w_2}{s_1w_1}-1=\frac{T_1-T_2}{t_1-t_2}-1=\frac{(T_1-t_1)-(T_2-t_2)}{t_1-t_2}$

Then, $$hA=\frac{s_2w_2}{\frac{(T_1-t_1)-T_2-t_2)}{t_1-t_2}}\log_e\frac{T_1-t_1}{T_2-t_2}$$

But, $$ht_mA=s_2w_2(t_1-t_2)=s_1w_1(T_1-T_2)$$

$$\therefore \quad t_m=\frac{s_2w_2(t_1-t_2)}{hA}$$

$$=\frac{s_2w_2(t_1-t_2)}{\left\{\frac{s_2w_2(t_1-t_2)}{(T_1-t_1)-(T_2-t_2)}\right\}\log_e\frac{T_1-t_1}{T_2-t_2}}$$

$$=\frac{(T_1-t_1)-(T_2-t_2)}{\log_e\frac{T_1-t_1}{T_2-t_2}} \quad \ldots\ldots\ldots\ldots \quad (4)$$

Case 4 as in Figs. 59 and 61.

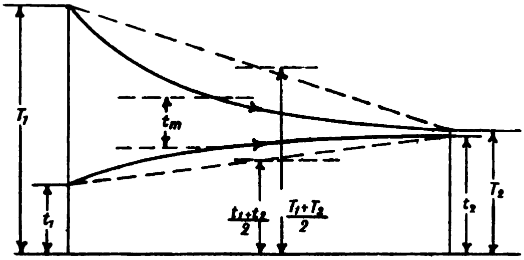


FIG. 59.—Temperatures of fluids with parallel-current flow.

Calculations similar to case 3 also leads to the result,

$$t_m=\frac{(T_1-t_1)-(T_2-t_2)}{\log_e\frac{T_1-t_1}{T_2-t_2}} \quad \ldots\ldots\ldots\ldots \quad (5)$$

An inspection of all these equations (1) to (5) for t_m shows that they are identical and may be stated thus,

Mean temperature difference=

$$\frac{\text{Initial temp. diff—Final temp. diff.}}{\log_e\frac{\text{Initial temp. diff.}}{\text{Final temp. diff.}}} \quad \ldots\ldots \quad (6)$$

In all the figures 56 to 59 the mean difference of temperature t_m is indicated. For approximate calculations it is sometimes assumed that the difference of the arithmetic means of inlet and outlet temperatures $\left(\frac{T_1+T_2}{2}-\frac{t_1+t_2}{2}\right)$ represents the mean difference of temperature, but it is seen from the diagrams that this may involve a large error, particularly in case 4 (parallel current flow).

The expression for the Reynolds law,

$$H=(A+B\rho v)\ (T-\theta)=\left(A+B\frac{w}{a}\right)(T-\theta)$$

can be more readily applied to boiler problems if modified somewhat for the convenience of calculation.

Using symbols similar to those given on p. 142, and letting θ_g and θ_w represent the wall temperatures on the gas side and water side respectively,

$$\text{Let } \left(A_1+B_1\frac{w_1}{a_1}\right)(T-\theta_g)=c_1\frac{w_1}{a_1}(T-a) \quad . \ . \ . \ . \ . \ . \ . \ . \quad (7)$$

$$\text{and } A_2+B_2\left(\frac{w_2}{a_2}\right)(\theta_w-t)=c_2\frac{w_2}{a_2}(a-t) \quad . \ . \ . \ . \ . \ . \ . \ . \ . \quad (8)$$

where a is an appropriate constant.

Expressions (7) and (8) may be utilised to obtain general equations in the following manner.

Counter Current Flow, Fig. 60.

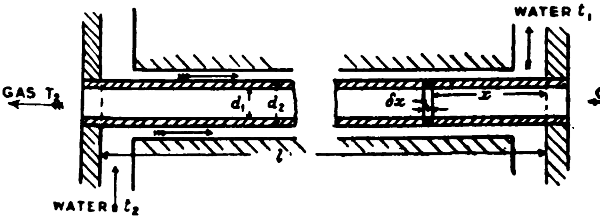


FIG. 60.—Counter-current flow.

$$\text{Heat lost by gas} \quad =\delta H=c_1\frac{w_1}{a_1}\ (T-a)\ \pi d_1\delta x \quad . \ . \ . \ . \quad (9)$$

$$\text{Heat gained by water}=\delta H=c_2\frac{w_2}{a_2}\ (a-t)\ \pi d_2\delta x \quad . \ . \ . \ . \quad (10)$$

where endwise conductivity along the tube is neglected.

$$\text{Also,} \qquad \delta H=-s_1w_1\delta T=-s_2w_2\delta t. \quad . \ . \ . \ . \ . \quad (11)$$

From (9) and (10)

$$\frac{T-a}{a-t}=\frac{c_2}{c_1}\times\frac{w_2}{w_1}\times\frac{a_1}{a_2}\times\frac{d_2}{d_1}$$

$$=nR\beta=r$$

where $$n=\frac{c_2}{c_1},\ R=\frac{w_2a_1}{w_1a_2},\ \beta=\frac{d_2}{d_1}$$

$$\therefore\ T-a=ra-rt$$

or $$a=\frac{T+rt}{r+1}$$

$$\therefore\ T-a=\frac{r}{r+1}(T-t)\ \ldots\ldots\ldots\ldots \tag{12}$$

also, $$a-t=\frac{1}{r+1}(T-t)\ \ldots\ldots\ldots\ldots \tag{13}$$

From (9), (11), and (12)

$$c_1\frac{w_1}{a_1}\pi d_1\frac{r}{r+1}(T-t)\ \delta x=-s_1w_1\delta T$$

or $$\frac{c_1}{s_1}\,\frac{r}{r+1}\,\frac{\pi d_1}{a_1}\,\delta x=-\frac{\delta T}{T-t}\ \ldots\ldots\ldots\ldots \tag{14}$$

From (11)

$$s_1w_1(T_1-T_2)=s_2w_2(t_1-t_2)$$

or $$\frac{s_2w_2}{s_1w_1}=\frac{T_1-T_2}{t_1-t_2}=K$$

$$=\frac{T_1-T}{t_1-t}=\frac{T-T_2}{t-t_2}$$

$$\therefore\ K(t_1-t)=T_1-T$$

or $$t=t_1-\frac{T_1-T}{K}.$$

Again, $K(t-t_2)=T-T_2$

or $$t=t_2+\frac{T-T_2}{K}$$

$$\therefore\ T-t=T-t_1+\frac{T_1-T}{K}$$

$$=\frac{KT-Kt_1+T_1-T}{K}$$

$$=\frac{K-1}{K}\left(T-\frac{Kt_1-T_1}{K-1}\right)$$

$$=\frac{K-1}{K}(T-b)\ \ldots\ldots\ldots\ldots \tag{15}$$

Substituting in (14) the value of $(T-t)$ from (15),

then,
$$\frac{c_1}{s_1}\frac{r}{r+1}\frac{\pi d_1}{a_1}\delta x=-\frac{K}{K-1}\frac{\delta T}{T-b}.$$

Integrating this,
$$\frac{c_1(K-1)}{s_1 K}\frac{r}{r+1}\frac{l}{m_1}=\log_e\frac{T_1-b}{T_2-b}\quad\ldots\ldots\ldots\quad(16)$$

Now from (15)
$$\frac{T_1-t_1}{T_2-t_2}=\frac{T_1-b}{T_2-b}$$

and since,
$$\frac{r}{r+1}=\frac{\frac{c_2}{c_1}R\beta}{\frac{c_2}{c_1}R\beta+1}=\frac{c_2R\beta}{c_1+c_2R\beta}$$

then, substituting in (16)
$$\frac{K-1}{s_1K}\frac{c_2c_1R\beta}{c_1+c_2R\beta}\frac{l}{m_1}=\log_e\frac{T_1-t_1}{T_2-t_2}\quad\ldots\ldots\ldots\quad(17)$$

Since $m_1=\frac{a_1}{\pi d_1}$, and $l\pi d_1=A_1$,

then
$$\frac{l}{m_1}=\frac{l\pi d_1}{a_1}=\frac{A_1}{a_1}.$$

Therefore equation 17 may be written .
$$\frac{A_1}{a_1}=\frac{l}{m_1}=\frac{s_1K}{K-1}\frac{c_1+c_2R\beta}{c_1c_2R\beta}\log_e\frac{T_1-t_1}{T_2-t_2}\quad\ldots\ldots\quad(18)$$

or, representing $\frac{s_1K}{K-1}\times\frac{c_1+c_2R\beta}{c_1c_2R\beta}$ by M . . . . (19)

then,
$$\frac{l}{m_1}=M\log_e\frac{T_1-t_1}{T_2-t_2}\quad\ldots\ldots\ldots\ldots\quad(20)$$

With certain modifications the above mathematical calculation is the same as that given by the late Professor J. T. Nicolson,* and for the sake of uniformity similar symbols have been used here. If the values of c_1 and c_2 are known, equation 18 conveniently expresses the relation between the surface of the tubes and the area for gas-flow in terms of temperatures, weight of gas and water-flow, and the internal and external diameters of the tubes.

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* "Boiler Economics and the Use of High Gas Speeds," *Trans. Inst. of Engs. and Shipbds. in Scotland,* Vol. LIV.

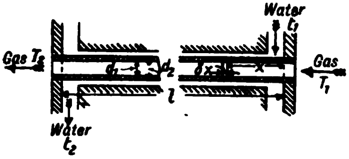

FIG. 61.—Parallel-current flow.

Parallel Current Flow.—In this case the gas and water flow in the same direction, as represented in Fig. 61, and the equations connecting the flow of heat become

$$\begin{aligned}\delta H &= c_1 \frac{w_1}{a_1}\left(T - a\right) \pi d_1 \delta x.\\ &= c_2 \frac{w_2}{a_2}\left(a - t\right) \pi d_2 \delta x.\\ &= -s_1 w_1 \delta T.\\ &= +s_2 w_2 \delta t.\end{aligned}$$

Making use of these equations in a similar manner leads to the following equations :—

$$\frac{A_1}{a_1} = \frac{l}{m_1} = \frac{s_1 K}{K+1} \frac{c_1 + c_2 R\beta}{c_1 c_2 R\beta} \log_e \frac{T_1 - t_1}{T_2 - t_2} \quad . \quad . \quad . \quad . \quad (21)$$

or, again,
$$\frac{l}{m_1} = M_1 \log_e \frac{T_1 - t_1}{T_2 - t_2} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Comparing this result with equation 18 for counter current flow it is seen that for given temperature conditions at inlet and outlet, $\frac{K}{K+1}$ will be less than $\frac{K}{K-1}$, and $\log_e \frac{T_1 - t_1}{T_2 - t_2}$ will be greater for parallel current than for counter current flow. Also it is evident that with parallel current flow the outlet temperature of the colder fluid could not possibly exceed the outlet temperature of the hot fluid.

The greater portion of the heat passing through a boiler heating surface is required to evaporate the water. Under such circumstances it may be assumed that the water is at a uniform temperature t and equation (14), p. 147, may be directly integrated, and becomes,

$$\frac{c_1 c_2 R\beta}{s_1(c_1 + c_2 R\beta)} \frac{l}{m_1} = \log_e \frac{T_1 - t}{T_2 - t}$$

or
$$\frac{A_1}{a_1} = \frac{l}{m_1} = \frac{s_1(c_1 + c_2 R\beta)}{c_1 c_2 R\beta} \log_e \frac{T_1 - t}{T_2 - t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

If c_1 and c_2 are not known the consideration of experimental results may be used to determine the value of $\frac{s_1(c_1+c_2R\beta)}{c_1c_2R\beta}=M$, say, which may then be employed for approximate boiler calculations.

Efficiency of a Boiler Tube.—Using the symbols explained on p. 142 the tube efficiency may be expressed as the ratio of the heat absorbed from the gases to the total heat in the gases measured from the temperature of the boiler water, taking the specific heat of the gases to be constant.

$$\text{Then, Tube efficiency}=\eta=\frac{(T_1-t)-(T_2-t)}{T_1-t}$$

$$=1-\frac{T_2-t}{T_1-t} \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (24)$$

According to the equation 23, p. 149, this may also be expressed as follows :—

$$\log_e\frac{T_2-t}{T_1-t}=-\frac{l}{m_1}\cdot\frac{c_1c_2R\beta}{s_1(c_1+c_2R\beta)}$$

$$\text{or,} \quad \frac{T_2-t}{T_1-t}=\epsilon^{-\frac{lc_1c_2R\beta}{m_1s_1(c_1+c_2R\beta)}}$$

$$\therefore \quad \eta=1-\epsilon^{-\frac{lc_1c_2R\beta}{m_1s_1(c_1+c_2R\beta)}} \quad . \quad . \quad . \quad . \quad (25)$$

The tube efficiency could only equal unity when $T_2-t=0$, i.e. when the exit temperature of the gases was the same as that of the boiler water. It may be inferred that with a tube of finite length this could only occur when $w_1=0$ and that the tube efficiency tends towards unity as w_1 approaches zero.

Some early experiments on heat transmission, of historical value only though sometimes quoted, made by Professor L. Ser, are described by him in his book, *Traité de Physique Industrielle*, Vol. I, 1888. One arrangement which he used consisted of a copper tube surrounded by a water jacket. Air was allowed to flow through the tube, entering at about ordinary atmospheric temperature and warm water was allowed to flow through the jacket. The inlet and outlet temperatures of the air and water were measured by mercury thermometers, but seeing that the temperature of the warm water only fell a fraction of a degree and that of the air only rose a few degrees in passing through the apparatus, errors of temperature measurement might have a large influence on the calculated results. It may be mentioned that the volume of air flowing was measured by the descent of a gasometer and verified by meter. Four tubes were experimented with of diameters 10, 20, 30, and 50 mm. (·254, ·508, ·762, and 1·97 in. respectively) and 1 metre (3·28 ft.) in length. Professor Ser did not tabulate the actual temperatures and volumes, but gave a series of values read off from mean curves he had obtained from plotted values. These results of the rate of heat transmission are shown in Fig. 62, converted into B.Th.Us. per square foot per second per deg. Fahr. difference between the water and the air, on the base representing approximately $\frac{w_1}{a_1}$. Apparently the rate of heat transmission

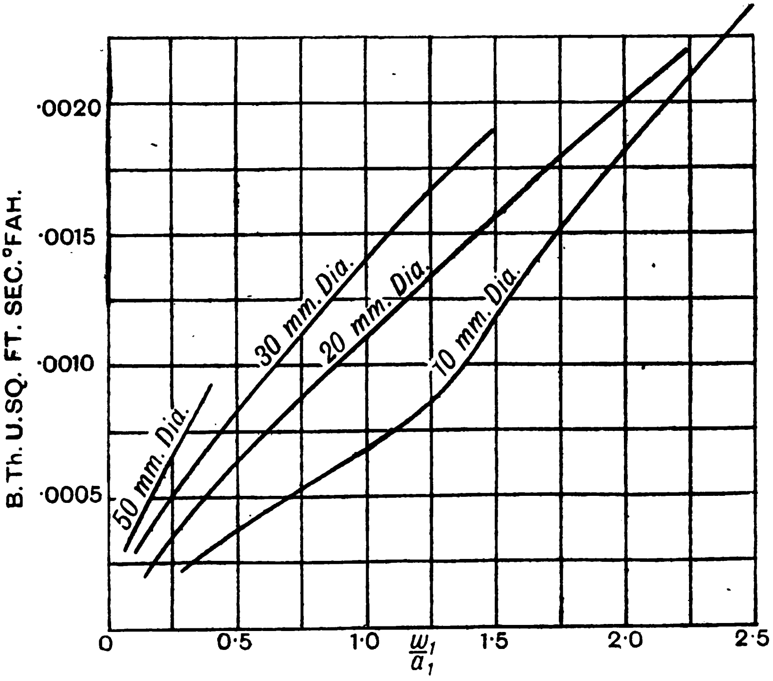

FIG. 62.—Rates of heat transmission obtained from Ser's experiments.

increased with the diameter to the extent shown in the figure, but such a result is extremely doubtful seeing that the general tendency of an increase of diameter is to reduce slightly the heat transmission rate when the velocity of flow exceeds the critical value.

Another series of experiments were made on a vertical cast-iron pipe 200 mm. (7·9 in.) internal diameter, thickness 8 mm. (·315 in.), and length 2·6 metres (8·5 ft.), placed inside a square cavity of brickwork having sides 380 mm. (15 in.). Warm water passed through the tube and the air along the space between the tube and the brickwork. The following table indicates the temperature conditions obtaining during the tests. Altogether sixteen tests are recorded on this apparatus.

VELOCITY OF AIR.		TEMPERATURES, °C.				Approx. value of $v_1\rho_1 = \frac{w_1}{a_1}$
Metres per sec.	Feet per sec.	Warm water.		Air.		
		Inlet.	Outlet.	Inlet.	Outlet.	
·678	2·22	79·0	71·0	1·0	22·9	·169
4·609	15·1	80·5	76·5	1·8	12·5	1·15

The rate of heat transmission is plotted on the base $\frac{w_1}{a_1}$ in Fig. 63 signified by the mean line (1).

Experiments were also made on a similar pipe 200 mm. (7·9 in.) internal diameter, having 26 external ribs or gills of depth 40 mm. (1·58 in.) and varying in thickness from 8 mm. (·315 in.) at the root to 2 mm. (·08 in.) at the top. The water and air temperatures were similar in their values to those enumerated above and hardly require further mention. The rate of heat transmission is shown in Fig. 63 also, the mean line (2) indicating the general relation.

A further series of experiments were made on a similar cast-iron pipe 250 mm. (9·8 in.) internal diameter having 50 ribs of depth 50 mm. (1·97 in.) and thickness varying from 8 mm. (·315 in.) at the root to 2 mm. (·08 in.) at the top. Two of these tubes were used, one surrounded by sheet iron, leaving a distance of 1 cm. (·394 in.) around the extremity of the ribs and giving a sectional area for the air flow of ·0488 sq. m. (·525 sq. ft.); the other was placed in a rectangular cavity leaving a sectional area for air-flow of ·098 sq. m. (1·05 sq. ft.). The air entered at about 13° C. and left at temperatures varying from 55° C. to 37° C. The results obtained are represented in Fig. 63 by the square points and line (4) for the first-mentioned arrangement, and by the triangular points and curve (3) for the second case.

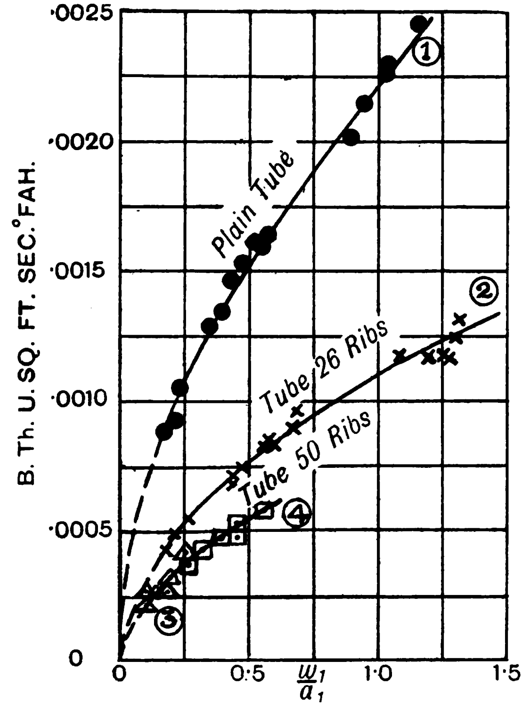

FIG. 63.—Rates of heat transmission obtained from Ser's experiments.

Some experiments have been made by Professor Josse, of Charlottenberg, on the transmission of heat to air at different pressures and rates of flow. The particular purpose of these experiments was to get some measure of the effect of air in surface condensers,* but here they are used to illustrate the applicability of the Reynolds law to conditions where there were large changes in the air pressure and corresponding changes of density. The apparatus was arranged as shown in the diagrammatic sketch, Fig. 64. It consisted of a pipe 1320 mm. (52 in.) long, of internal diameter 23 mm. (·906 in.), surrounded by a steam jacket which was kept at 100° C. The volume of air passing through was measured by a meter, and the air pressure was indicated by a vacuum gauge; the average temperature at the air inlet to the tube was about 20° C. (68° F.). A series of experiments were made at the three pressures, 1·034, ·515, and ·106 atmospheres respectively. In Professor Josse's paper the results of the calculated rates of heat transmission were plotted on a velocity of air base and three different curves resulted. The author has calculated the weight of air-flow for each test and plotted the rate of heat transmission in British units in Fig. 65 on the base $\frac{w_1}{a_1}$.

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* "Surface Condensers for Steam Turbines," *Engineering*, Dec. 11th, 1908.

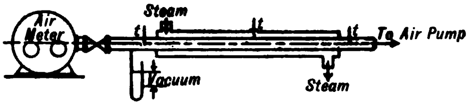

FIG. 64.—Arrangement of heat-transmission apparatus used by Josse.

The different character of the points in Fig. 65 indicates the results at the different pressures, and it is clearly seen that they all lie near a mean curve in much the same manner as in previous experiments at different pressures discussed on pp. 125 and 126, and again verify Reynolds law within the limits of the experiments. Using the equation 23, p. 149, the values of M were calculated from each of the test results and are also shown plotted in Fig. 65.

Following up the work of Professor Osborne Reynolds the late Professor J. T. Nicolson has done a large amount of experimental work on heat transmission, particularly with reference to the applicability of the Reynolds law to the design and working of boilers. Some of his experiments were conducted on a large scale and these are described in detail in *Heat Transmission in Boilers, Condensers, and Evaporators.*

Some of the earlier experiments made by Professor Nicolson are described briefly in a lecture on "The Laws of Heat Transmission in Steam Boilers, as Deduced by Experiment."* A part of the apparatus consisted of an arrangement for cooling compressed air by means of a surface cooler at the works of Messrs. Joseph Adamson and Company, Hyde. The air was cooled by passing down an annular space between two concentric tubes, up the inner of which the cold water was made to flow. The bore of the inner tube was ⅜ in., having an outside diameter ·516 in., and the inside diameter of the outer pipe was ·898 in. The length was about 12·5 ft.

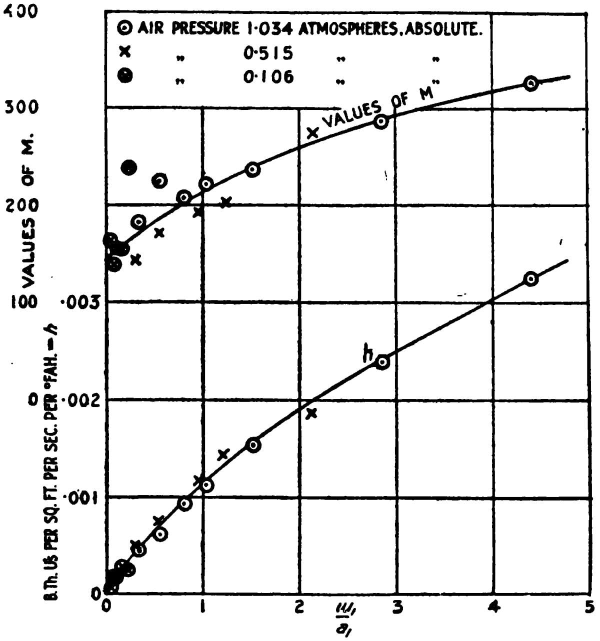

FIG. 65.—Rate of heat transmission between air and water at various air pressures, from Josse's experiments.

The pressure of the air at the entry to the cooling pipe was about 60 lb. per sq. in. by gauge, and the pressure dropped 10 to 20 lb. per sq. in., while passing through the annular pipe. Its inlet temperature was from 100° F. to 160° F. and the outlet varied from 70° F. to 90° F. The air was probably saturated with moisture, and any water thrown down in the collecting vessel at the bottom could be drawn off through a tap and measured. The dryness of the air leaving the collecting vessel was observed by wet and dry bulb thermometers placed in a wooden box (with glass front) through which a small quantity of the air was allowed to flow and to escape through a hole in the top of the box. The whole apparatus was erected in the hottest part of the works' power station, and as the temperature of the surrounding air was usually over 90° F., the amount of radiation from the outside of the air pipe would be so small as to be almost negligible.

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* *Junior Inst. of Engs.*, 1908.

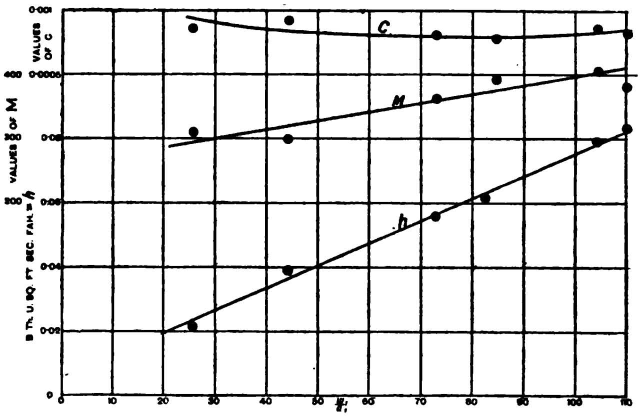

FIG. 66.—Rate of heat transmission, from Nicolson's experiments.

The cold water was taken from a large overhead feed tank at a temperature of from 60° F. to 80° F., according to the time of the year, and it was discharged into a measuring tank fitted with a stilling bay and water gauge glass. The weight flowing was measured by a circular sharp-edged orifice in the bottom of the tank and the head of water read from the gauge glass. The rate was frequently checked by catching the flow for a measured interval and weighing it. A selection of the results of the experiments was given in the paper from which the observed data given in Table 17 are obtained.

The rate of heat transmission *h* in column 10 is shown plotted in Fig. 66, giving practically a straight line. Using

TABLE 17

Water channel, circular, bore ⅜ in.; a_2= ·000766 sq. ft.; cooling surface, 1·225 sq. ft.

Air channel, annular, diameters of pipes, ·516 and ·898 in.; a_1= ·0029 sq. ft.; length, 12·5 ft.; heating surface 1·689 sq. ft.; $\frac{l}{m_1}$ = 582 for heat-flow.

No. of Expt.	Air temperatures.		Water temperatures.		w_1 and $\frac{w_1}{a_1}$	w_2 and $\frac{w_2}{a_2}$	B.Th.U. per sq. ft. per sec. (air side).	Mean temperature difference °F.	B.Th.U. per sq. ft. (air side) per sec. per °F. difference air to water h.	Value of M.
	Entering T_1 °F.	Leaving T_2 °F.	Entering t_2 °F.	Leaving t_1 °F.						
1	2	3	4	5	6	7	8	9	10	11
1	133	72	58·5	65·6	·211 72·8	·431 563	1·87	33·5	·0559	362
2	109	69	62·2	64·5	·074 25·5	·306 402	·429	20·05	·0214	309
3	135	70	60·0	64·9	·1275 44·0	·401 524	1·195	30·9	·0387	299
5	137	75	59·4	68·5	·245 84·6	·397 519	2·2	35·7	·0617	392
6	154	94	79·9	89·0	·319 110	·498 653	2·78	33·3	·0835	381
7	163	96·5	80·0	93·8	·302 104	·346 452	2·91	36·7	·0793	406

equation 20, p. 148, the various values of M were calculated and are given in column 11 and are also shown plotted in Fig. 66 along with the values of c in the relation,

$$h=c\frac{w_1}{a_1}.$$

The experiments made by Mr. Jordan were discussed on pp. 127 to 139, having regard principally to their bearing on the Reynolds law of heat transmission. Without reference to the wall temperature the rate of heat transmission h between the air and the water is shown plotted in Fig. 67 on the base $\frac{w_1}{a_1}$, the mean difference of temperature being calculated by the formula 6 on p. 145. Comparing the series D, E, F, in Fig. 67 indicates that reducing the value of m_1 causes an increase in the rate of heat transmission.

The values of M from equation 20, p. 148, are also plotted in Fig. 67, and also the values of c_1 derived from equation 19, p. 148. In obtaining these values of c_1 use was made of the mean line of Fig. 81, p. 180, which was taken to give the probable values of $C_2=c_2\frac{w_2}{a_2}$. Both the values of M and c_1 appear to be somewhat erratic, but this is probably due to some extent to the experimental conditions.

An elaborate series of experiments have been made on small model boilers by H. Kreisinger and W. T. Ray.* The apparatus is shown in Fig. 68 and consisted mainly of small horizontal tubular boilers which were interchangeable and which were supplied with air heated in an electric furnace. The steam generated in the boiler was condensed in a small surface condenser and the quantity accurately determined. A steam ejector, connected to the exit end of the boiler, caused a current of air to flow through the boiler and furnace. The box containing the boiler and furnace was made of " asbestos slate " and the annular space around the boiler was filled with mineral wool and magnesia covering.

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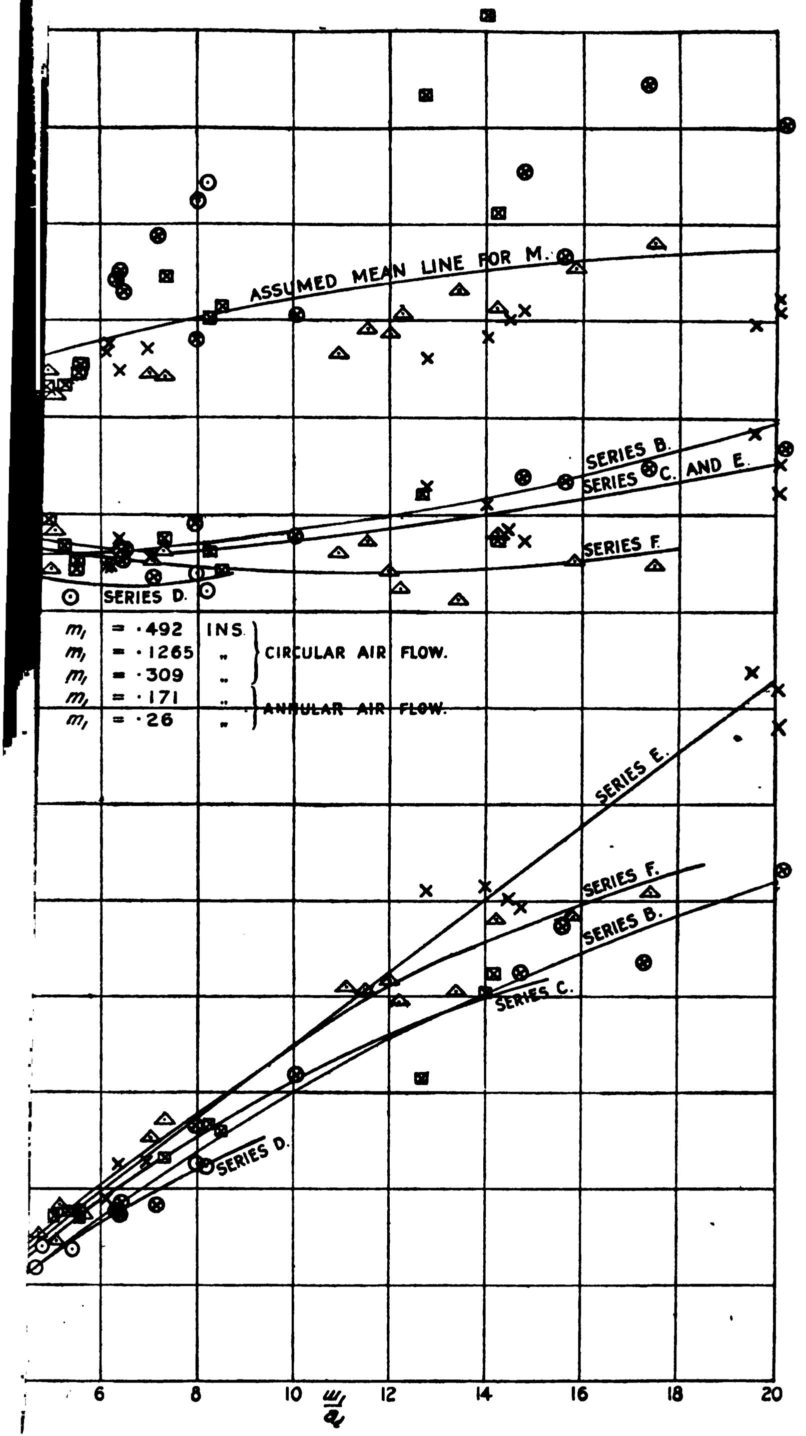
* *The Transmission of Heat into Steam Boilers,* Bulletin 18, Department of the Interior, Bureau of Mines, Washington.



Rate of heat transmission, from Jordan's experiments.

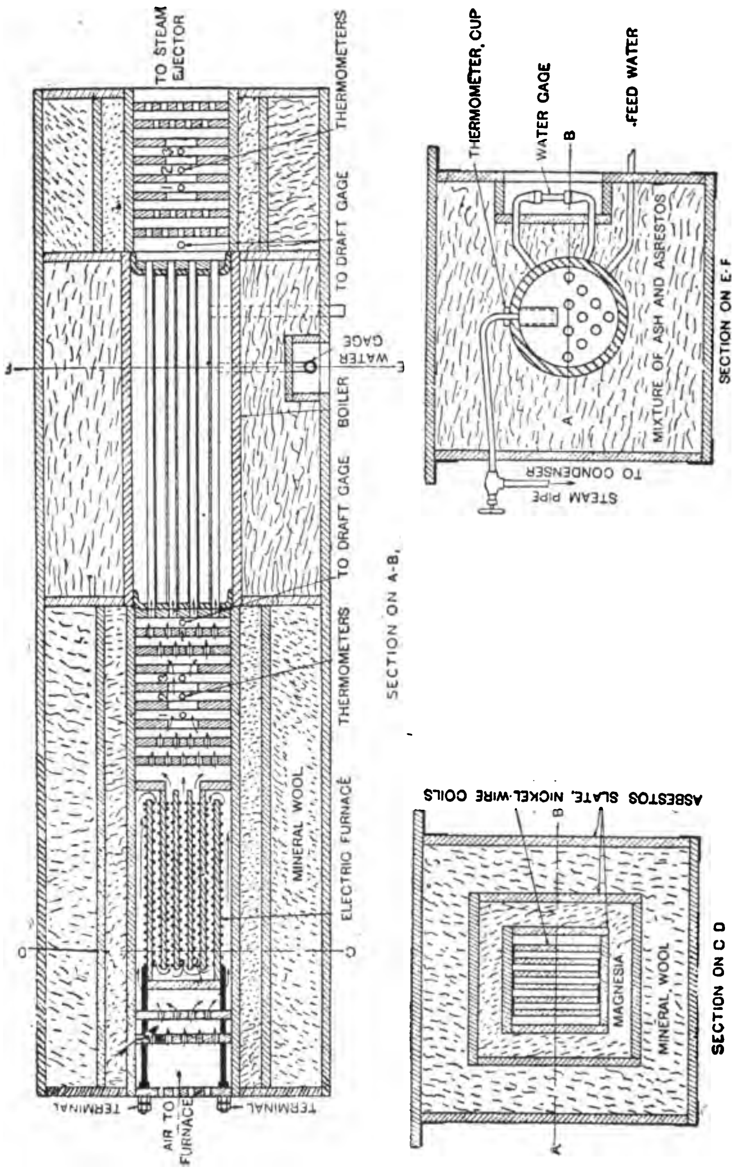


FIG. 68.—Small boilers used in Kreisinger and Ray's experiments.

The Boilers.—Experiments were made with four horizontal boilers having the dimensions given in Table 18.

TABLE 18

Number of boiler	1	2	3	4
Length of boiler, outside to outside of boiler ends, inches l	8·28	8·28	16·125	8·28
Number of copper tubes	10	10	10	10
Actual outside diameter of tubes, inches	·252	·313	·252	·3745
Actual inside diameter of tubes, inches	·175	·230	·175	·2928
Total area of cross section of tubes, square feet $= a_1$	·00167	·002885	·00167	·00467
Total heating surface (gas side) of tubes, square feet	·3162	·4155	·6156	·529
$\frac{l}{m_1}$ for heat-flow	189	144	369	113
$\frac{l}{m_1}$ for resistance to air-flow	200·5	153	380	120

The boilers were constructed of a piece of standard 4-in. wrought-iron pipe for the outside shell of each boiler. The boiler ends were made of $\frac{1}{8}$-in. sheet copper and were riveted to the 4-in. pipe. All the boilers had ten tubes of thin copper, which were expanded and soldered into holes in the boiler ends, the positions being indicated by the sections in Fig. 68. Both boiler ends were covered with $\frac{1}{4}$-in. asbestos paper and slate so that only the heating surface of the tubes was really effective. This is the reason for the comparatively large difference in the values of $\frac{l}{m_1}$ for heat-flow and for resistance to the air-flow shown in the above table. Each boiler was fitted with a water-gauge glass, a thermometer cup for measuring the temperature of the steam, a pipe for feeding the boiler, and a pipe for letting out the steam. The boiler was fed with distilled water obtained by condensing steam from the main boiler of the plant.

The electric furnace is illustrated in two sectional views in Fig. 68. It consisted of six coils of pure nickel wire wound around rectangular pieces of asbestos slate, $\frac{9}{32}$ in. thick, $3\frac{7}{8}$ in. wide, and 12 in. long. The nickel wire was ·072 in. diameter, about No. 13 B. and S. gauge, there being 51 turns in each coil, so that the total length of the wire in the furnace was 204 ft. In most of the tests the coils were connected in series; with high temperatures and high pressure drops or "draughts" three coils were connected in series, and the two sets were connected in parallel. A water rheostat was used for adjusting the electrical energy to each temperature and draught. The coils were placed vertically in the front portion of the asbestos slate box with a space of about $\frac{5}{16}$ in. between them, and were so baffled that the air passed along the coils three times before it left the furnace. Direct current of 220 volts was used and the energy was measured by an integrating wattmeter.

With temperatures up to about 900° F. there was no particular trouble, but when the temperature of the air was raised to 1200° F. and 1500° F. the nickel wire seemed to crystallise and become very brittle after cooling. The coils usually failed by fusing after they had been in use ten hours a day for six to ten days. It must be understood that in order to raise the temperature of the air to 1500° F. the coils were sometimes heated close to the point of fusion (2600° F.).

The space between the furnace and the boiler was fitted with eight perforated screens, and the four middle screens had in their centres round $1\frac{1}{4}$-in. holes. In the spaces between these four screens three thermometers were so inserted that their bulbs were in or near the centre line of the $1\frac{1}{4}$-in. holes, and an inspection of Fig. 68 enables the arrangement to be understood. The object of these screens was to confine the air around the bulbs of the thermometers in order that its temperature could be more accurately measured. The two screens nearest the furnace and the two nearest the boiler each had thirty-two $\frac{1}{4}$-in. holes, so drilled that the thermometers were perfectly screened from direct radiation from either the boiler or the furnace. Thus the enclosure in which the thermometers were situated had all the surfaces at practically the same temperature as the thermometers, and therefore the bulbs should have the same temperature as the air.* A similar arrangement was employed for obtaining the temperature of the air leaving the boiler. Tubes for measuring the pressure of the air were inserted on both sides of the boiler between the boiler end and the first screen.

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* Refer to p. 18, for a discussion as to the importance of this matter at high temperatures.

The asbestos-slate box forming the body of the apparatus was 1 ft. high by 1 ft. wide by 5 ft. long. The spaces enclosing the furnace and behind the boiler were 4 in. square, so that the total thickness of the insulating walls was 4 in. The walls around the furnace consisted of a ¼-in. lining of asbestos slate, and a 1-in. layer of magnesia boiler covering. The space between the layer of magnesia and the outside sheet of the box was filled with mineral wool. The space around the boiler was filled with a mixture of wood ashes and asbestos.

The temperatures up to 700° F. were measured with mercurial thermometers, which were compared with a standard thermometer calibrated by the Bureau of Standards in Washington. Temperatures above 700° F. were measured by calibrated platinum, platinum-rhodium thermo-couples directly exposed to the hot air.

Although great care was taken in placing and in screening the thermo-couples, as was described on p. 161, considerable difficulty was experienced in measuring the initial and final temperatures of the air. With the initial temperatures of 1200° F. and 1500° F., the three thermo-couples would at times read as much as 100° F. apart, and for no apparent reason. Generally the thermo-couple closest to the furnace read higher than the other two. The only reasonable explanation of this fact was that in spite of the screens this thermo-couple received some heat by radiation from the furnace, whereas the one nearest the boiler lost some heat by radiation to the boiler. The thermometers on the side of the leaving gas read from 10° F. to 20° F. apart. It was difficult to come to any conclusion as to which of the thermometers or thermo-couples were wrong, and it was thought best to take the average of the three readings at either end. These results illustrate the great difficulties experienced in attempting to measure accurately high temperatures of a moving gas.

Before starting a test the apparatus was brought to the temperature at which the test was to be run, and after a test was started all conditions were kept as nearly uniform as possible. The duration of each test was from thirty minutes to two hours. The higher the draught and the temperature the shorter was the duration of the test made.

In all but a few of the experiments the steam was generated at atmospheric pressure, the steam temperature being in the neighbourhood of 212° F., and the feed water temperature varied in different tests from about 62° F. to about 144° F., but generally it was in the neighbourhood 70° F. to 100° F.

The weight of air w_1 passing through the boiler in pounds per second was calculated from the weight of steam condensed and the heat given per pound of steam. It was taken that the steam was without moisture as it left the boiler.

If W=weight of steam condensed, lbs. per sec.
t_s=temperature of steam, °F.
t_w=temperature of feed water, °F.
T_1=air temperature at inlet, °F.
T_2= „ „ „ outlet, °F.

Then w_1 was estimated as follows :—

$$W\{965{\cdot}7+(t_s-t_w)\}={\cdot}2375w_1\,(T_1-T_2)$$

$$\text{or, } w_1=\frac{W\{965{\cdot}7+(t_s-t_w)\}}{{\cdot}2375\,(T_1-T_2)}$$

Altogether the results of 237 tests are tabulated in the bulletin, and are too numerous to reproduce here in tabular form. Table 19, however, indicates the conditions of the experiments sufficiently well for the present purpose. The values in column 3 represent the average air inlet temperatures during each of the series of tests. In columns 4 and 5 are given respectively the maximum and minimum mean air inlet temperatures in each series, each value, of course, representing the average reading of the three thermometers in the particular tests. Similarly, columns 6 and 7 contain respectively the maximum and minimum mean air outlet temperatures. In each of the series the principal independent variable condition was the draught and the consequent variation of the rate of flow of the air through the tubes. This is subsequently indicated in the various graphs, so that a tabulation of the rate of air-flow is hardly necessary.

TABLE 19

Boiler Number.	No. of tests in series.	Air inlet temperatures.			Air outlet temperatures.		Steam temperatures °F.	Draught, inches water.		Coefficient * of tube resistance *f*.
		Average in each series. °F.	Maximum in each series. °F.	Minimum in each series. °F.	Maximum in each series. °F.	Minimum in each series. °F.		Highest in the series.	Lowest in the series.	
1	2	3	4	5	6	7	8	9	10	11
1.	19	497	504	489	268	223	209 to 211	16·9	·3	·019
	19	695	702	677	320	266	209 to 211	13·8	·5	·0142
	12	982	998	965	379	292	210	14·0	·5	·009
2.	27	495	511	481	313	243	210 and 211	22·9	·1	·0396
	16	675	684	658	378	295	210 to 212	24·0	·2	·0239
	29	900	957	860	482	324	mostly 211 to 216 a few 242 to 277	15·1	·1	·0154
	28	1217	1255	1176	626	374	211 to 227	23·6	·1	·0099
	11	1519	1540	1492	696	454	211 to 223	7·0	·3	·0069
3.	19	684	688	677	289	231	210 to 216	19·7	·5	·0088
	14	884	912	867	312	232	211 to 214	21·3	1·0	·0088
	10	1212	1232	1195	370	268	211 to 218	20·5	·85	·0071
	11	1511	1542	1485	404	245	211 to 222	20·1	·5	·006
4.	10	691	697	680	434	391	211 to 214	14·1	·5	·021
	9	898	913	880	509	434	211 to 216	13·2	·5	·0253
	3	1195	1207	1180	602	529	211 to 213	1·2	·5	·0153

* Calculated as on p. 169, after allowing for the pressure drop required to create the kinetic energy of flow at inlet to the tubes but neglecting any losses at inlet or recovery at outlet.

It would be evident from the description of the apparatus and of the conditions of the tests that great precautions were taken by the experimenters to ensure a reasonable degree of accuracy in their measurements.

The author has calculated, from the experimental values tabulated in the bulletin, the rate of heat transmission h in B.Th.U's. per square foot (air side) per second per deg. Fahr. difference between the air and the water, and the respective values of

$$\frac{w_1}{a_1}=\frac{\text{air-flow, lbs. per sec.}}{\text{area of flow, sq. ft.}}$$

The mean difference of temperature was given by

$$\frac{T_1-T_2}{\log_e\left(\dfrac{T_1-t}{T_2-t}\right)}$$

where t was taken to be a few degrees below the steam temperature, to allow for the low feed-temperature. The rates of heat transmission h were plotted on the base $\frac{w_1}{a_1}$ for each boiler, and Fig. 69 has been reproduced for boiler No. 2 to show the character of the results obtained. The whole of the mean lines, however, were reproduced in Fig. 70, without showing the plotted points, and these indicated the straight line relation connecting the rate of heat transmission h with $\frac{w_1}{a_1}$ within the limits of the experiments. The average inlet temperatures of the air were indicated on the lines, and it would be noticed that the temperature of the air had but little influence upon h. For some reason the values for boiler No. 3, which had tubes nearly twice the length of those in the other three boilers, were substantially lower than the others. An inspection of the results for boilers Nos. 1, 2, and 4, would also indicate that the rate of heat transmission h increased as the diameter of the tube decreased.

The calculated tube efficiencies

$$\frac{T_1-T_2}{T_1-t}$$

for boiler No. 2 are also shown plotted in Fig. 69.

To analyse further the results of these experiments the author calculated the various values of M and c_1 by means of

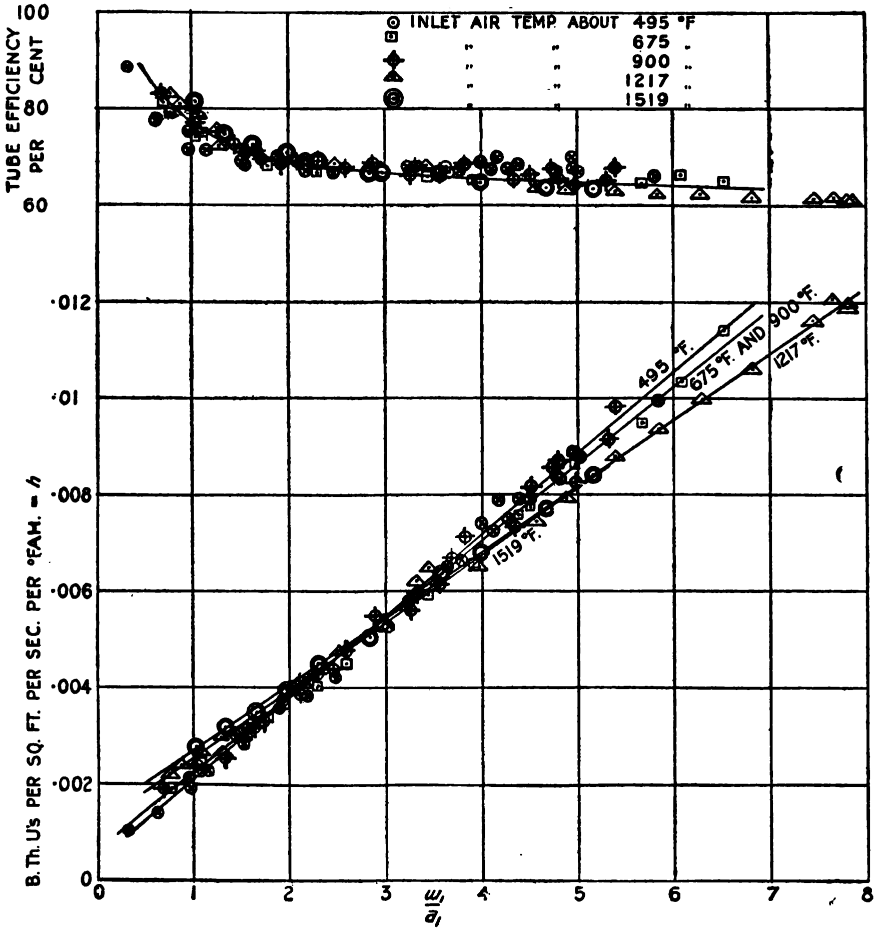

FIG. 69.—Rate of heat transmission for boiler No. 2, from Kreisinger and Ray's experiments.

the equation 23, p. 149, and using the value for $c_2\frac{w_2}{a_2}$* $=C_2=\cdot 03$ for boiler conditions on the water side of the tubes. The results obtained are indicated in Fig. 71 by the various mean curves, and as the actual plotted points were found to lie fairly close to the respective mean lines, it was not thought to be worth while to reproduce the points.

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* The conditions of circulation when water is boiling are not the same as when water is being merely heated and rising in temperature. Provided that large bubbles of steam are not allowed to cling to the heating surfaces due to

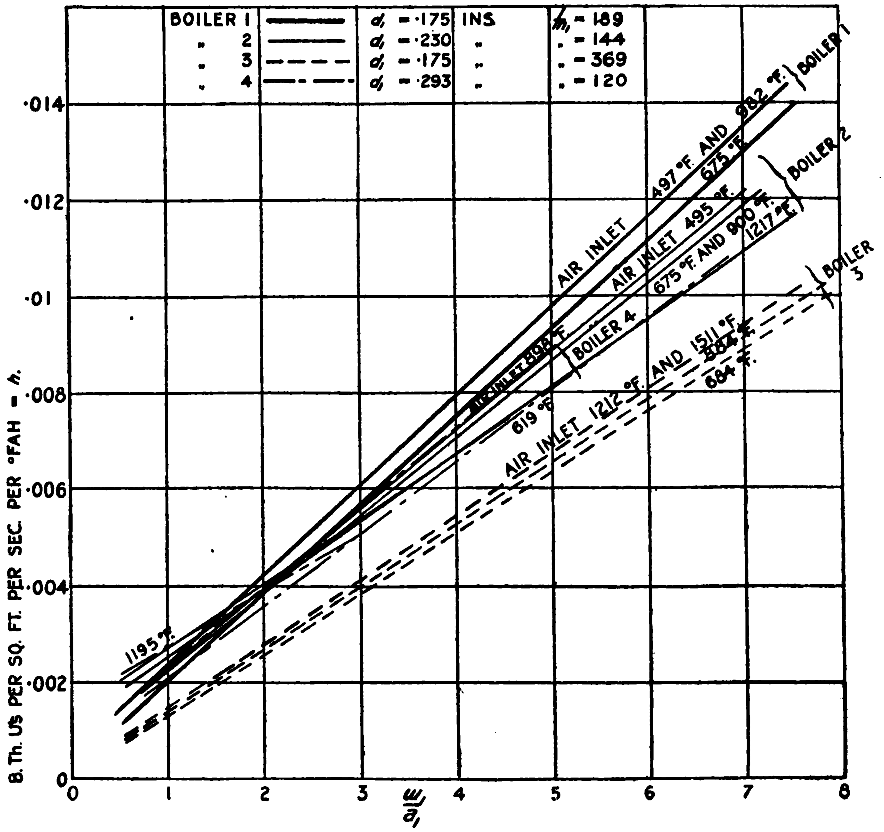

FIG. 70.—Rates of heat transmission from Kreisinger and Ray's small boiler experiments.

On referring to Fig. 68, p. 159, it would be seen that draught connections were made at both the inlet to and outlet from the

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bad circulation of the water, it is probable that the improvement of circulation due to the presence of steam bubbles lowers the resistance to the heat-flow on the water side. The value of $c_2\frac{w_2}{a_2} = C_2 = \cdot 03$ instead of $C_2 = \cdot 0131$, given on p. 180 is intended to allow for this reduced resistance.

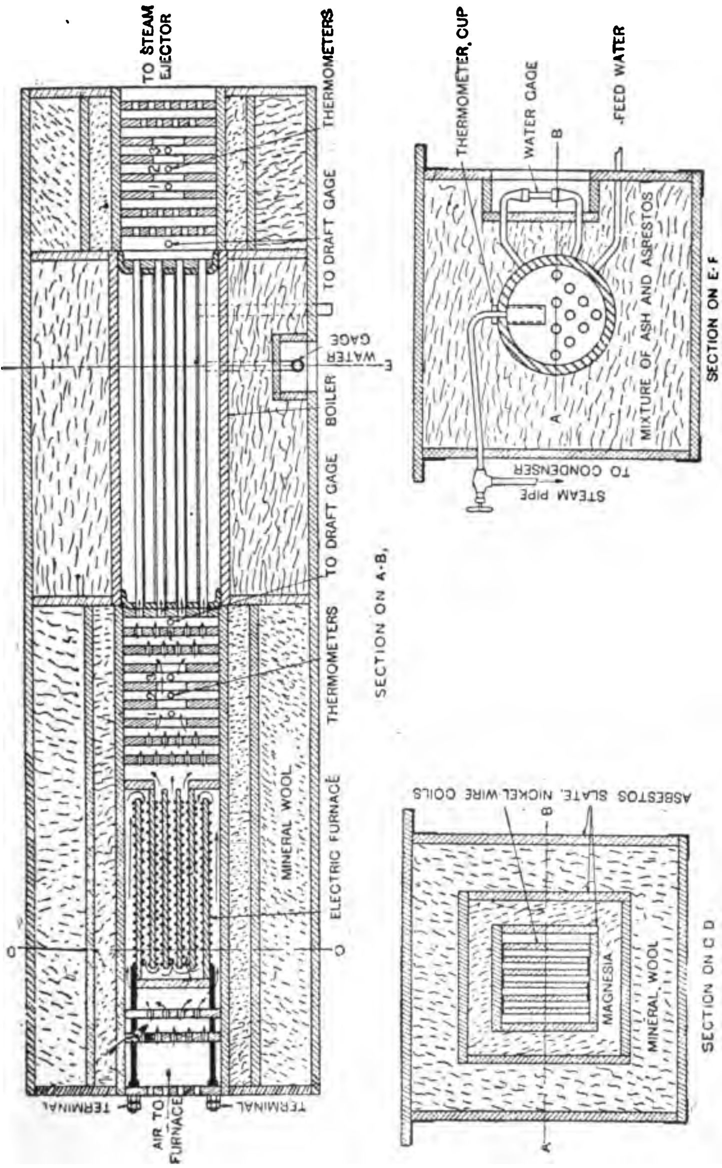

FIG. 68.—Small boilers used in Kreisinger and Ray's experiments.

plotted against log $\left(\frac{w_1}{a_1}\right)$. Mean lines were then drawn in at a gradient 2. As an example of the results obtained Fig. 72 is reproduced for Boiler No. 2. Although in some cases the plotted points lay about the mean lines in an erratic fashion, the author was able to calculate approximately the various values of the coefficient of resistance f, which are shown plotted in Fig. 73 against the air inlet temperatures, and are tabulated in column 11 of Table 19.

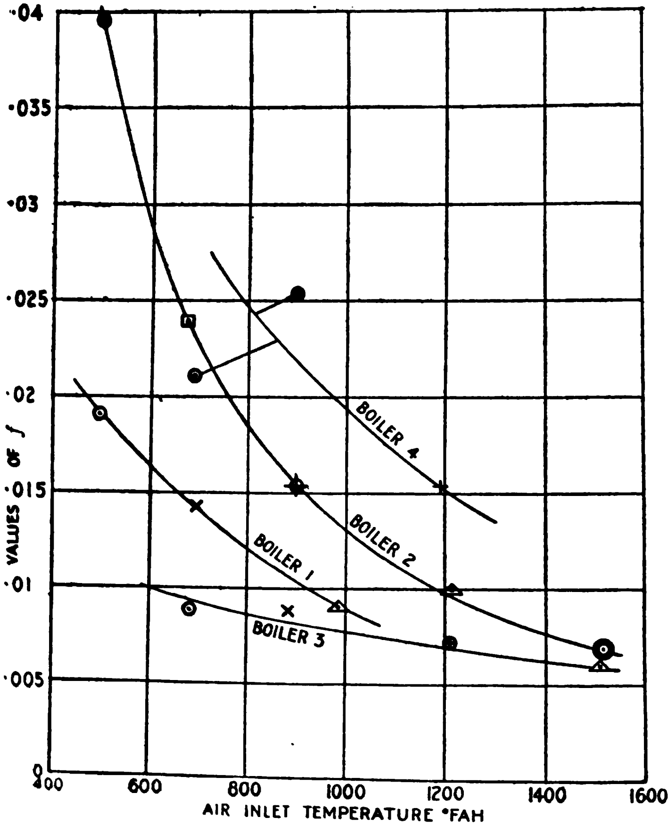

FIG. 73. -Coefficient of resistance to flow of gas, f, from Kreisinger and Ray's small boiler experiments.

The reasons for the rapid decrease of f as the temperature of the air increased is not at first obvious, but on consideration it would be clearly seen that values of f derived from experiments made at practically constant temperature could *not* strictly be applied to boiler conditions, for, in the latter case, the temperature of the air or gas decreases rapidly along the tube, and consequently the velocity decreases nearly in proportion to the absolute temperature. Thus there must be a continual decrease of momentum along the tube, and this change of momentum assists the air to overcome the resistance to the flow through the tubes. The higher the inlet temperature the greater does the decrease of momentum become, all other things equal, and consequently it has greater influence on the coefficient f. Also, with smooth pipes, it generally occurs that f decreases as the velocity of flow increases, and an increase of temperature means an increase of velocity for a given weight passing through. In any case, a comparison with the author's results given on pp. 182, 187, and 198, indicates a considerable difference in the values of f for no very evident reason.

The results of some experiments made by J. E. Bell are quoted in a paper on "The Steam Boiler of 1915."* The apparatus used consisted of a 2-in. inside diameter copper pipe, surrounded by 20 individual water jackets each 1 ft. long. The gases were drawn through the tube from a coal gas furnace in which temperatures above 2600° F. could be obtained. The difference of temperature between the gas and metal surface varied from 400° F. to 2000° F., and the temperature of the metal surface varied from 145° F. to 215° F., the average being 180° F. In the calculations the variation of the specific heat of the gases was taken into account. The flue gases, after leaving the experimental tube, were cooled in a 2-in. coil 26 ft. long surrounded by water, and were then led through a box where the dew point was determined. The flue gases were analysed, and this, together with the dew point and the measured temperatures of the gas and water at entrance to and at exit from the cooler coil, gave a measure of the weight of gas-flow.

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* Mechanical Engineering Section, International Engineering Congress, San Francisco, 1915.

The rates of heat transmission were shown in the paper as a series of straight lines similar to those shown in Fig. 74, and without plotted points, the experiments extending over the range of $\frac{w_1}{a_1}$ shown by these lines. The only differences between Fig. 74 and the original is in the scales adopted to show the rate of heat transmission and $\frac{w_1}{a_1}$. When values are

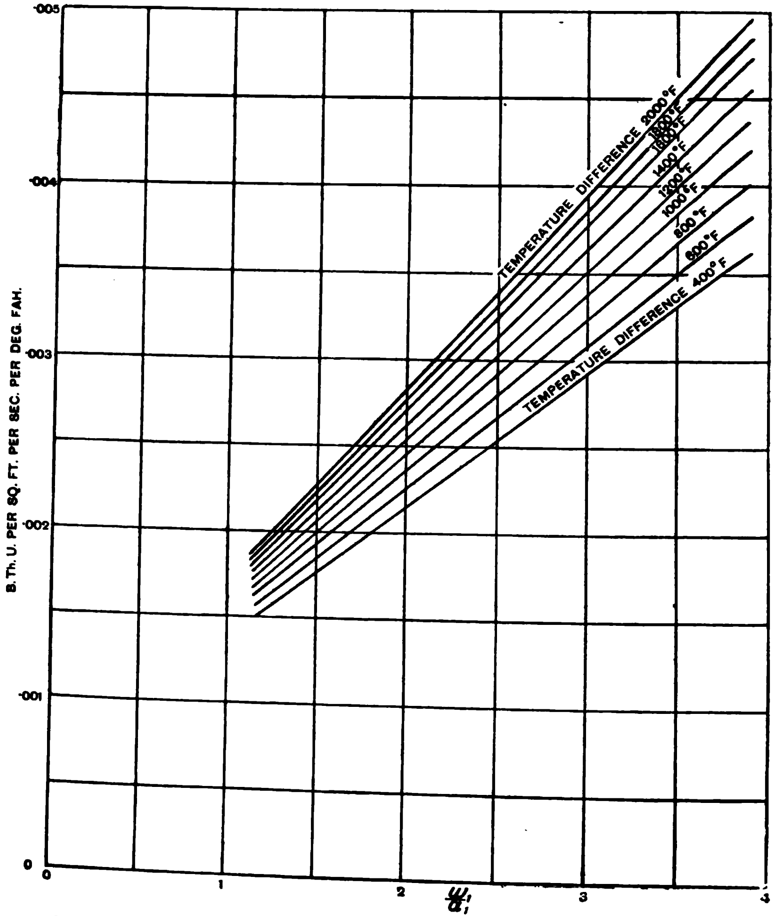

FIG. 74.—Rate of heat transmission, from Bell's experiments.

compared with those from all the other experiments on small apparatus and from the boiler experiments in *Heat Transmission in Boilers, etc.*, they show a remarkable degree of consistency.

A series of ten tests have been made by W. H. Carrier* and F. L. Busey on an air heater consisting of eight sections of 1-in. pipes, each section having four rows of pipes arranged as shown in Fig. 75. The air was drawn through the heater by means of an induced draught fan. Steam was supplied to the inside of the pipes through a separator at an absolute pressure of 19·7 lb. per sq. in., measured by a calibrated gauge, the corresponding saturated steam temperature being

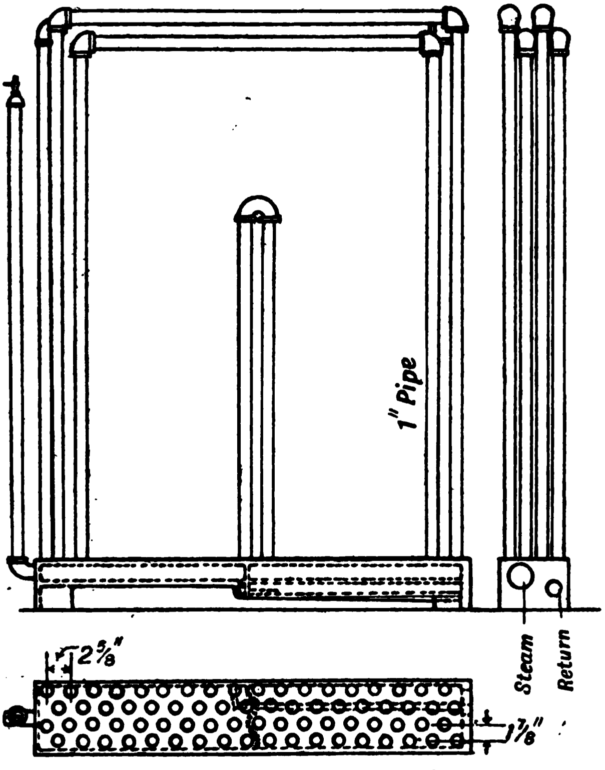

Fig. 75.—Heaters used in Carrier and Busey's experiments.

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* "Air Conditioning Apparatus," *Trans. Amer. Soc. Mech. Engs.*, Vol. 33, 1911, p. 1613.

227° F., and it was found that the steam supply at inlet was slightly superheated. The condensation from each section was weighed separately, and each section was provided with an air vent and thoroughly blown out for about one hour before starting a test. Also some steam was allowed to escape at the vent throughout the test to ensure uniform conditions on the steam side of the tubes. The average velocity of the air was measured at the fan outlet by means of a Pitot tube and the volume and weight of air calculated. It was found, however, that this calculated quantity of air was some 8% in excess of that estimated from the heat given up by the steam in condensing, and this latter method (see p. 163) has been adopted to give a measure of the weight of air-flow, without making any allowance for radiation loss, etc., from the heater casing. The thermometers for measuring air temperatures were protected from the effect of radiation. The inlet temperature of the air had an average of about 20° F., and in the first test at a low rate of air-flow the rise of air temperature was 157·9° F. and at the highest rate of air-flow the rise was 122·8° F.

The heating surface in contact with air in each section was 126·6 sq. ft., giving a total surface of 1010 sq. ft. The writer has estimated the average area available for air-flow among the tubes at 11·6 sq. ft. by the method explained on p. 76 of *Heat Transmission in Boilers, etc.*, and has calculated the rates of heat transmission from the steam to the air,* but without allowing for any heat taken up by the air from the surface of the casing. The results are shown plotted in Fig. 76 on the base $\frac{w_1}{a_1}$, and comparing with the values of h obtained from the experiments previously and subsequently discussed it is seen that those in Fig. 76 are much higher in value than the others, except for the small boiler experiments, Fig. 70.

These results therefore suggest that when the flow of gas is across tubes set zigzag fashion, the rate of heat transmission for a given value of $\frac{w_1}{a_1}$ is appreciably greater than when the gas flows through tubes. It would be noted, however, that under ordinary conditions of operation in practice, where steam would not be allowed to blow continuously, the rate of heat transmission would probably be lower than is represented by the results in Fig. 76, because the air in the steam would not be so freely discharged.

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* Under the conditions of these tests the temperature of the tubes would be nearly the same as the steam saturation temperature.

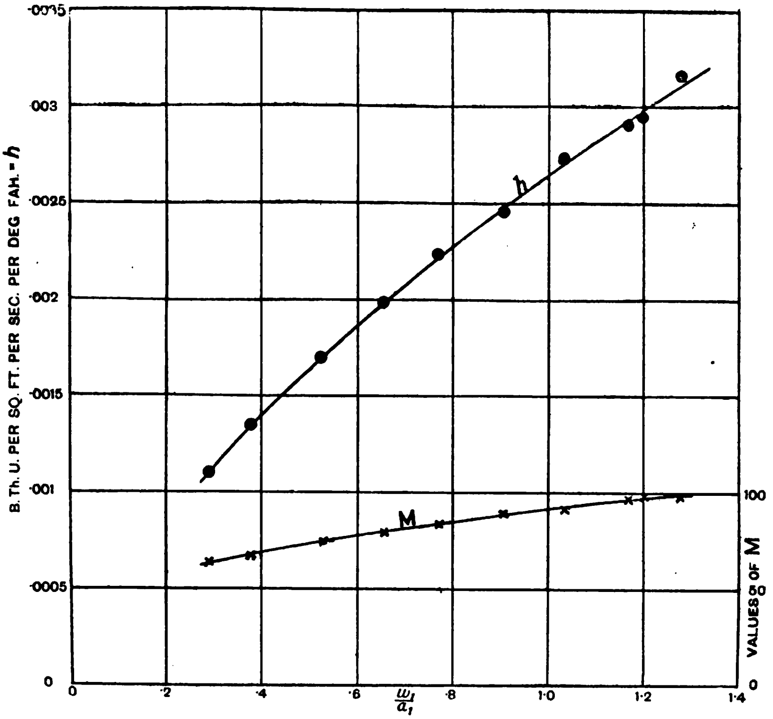

FIG. 76.—Rate of heat transmission, from Carrier and Busey's experiments.

Values of M in the equation—

$$\frac{A}{a_1}=\frac{l}{m_1}=M\log_e\frac{T_s-T_1}{T_s-T_2}.$$

have been plotted also in Fig. 76, where T_s is the condensing temperature.

It is now proposed to consider some experiments made by the author in the mechanical engineering laboratory at The

Royal Technical College, Glasgow.* One apparatus is illustrated in Fig. 77, and consisted of a solid drawn steel tube ·994 in. inside and 1·242 in. outside diameter, enclosed in a tube $1\frac{17}{32}$ in. inside diameter, the lengths and arrangement being as shown in the illustration.

Air under a small pressure was passed through the gas heater and then allowed to flow through the tube, the inlet and outlet temperatures being measured by mercury thermometers inserted with the bulbs in direct contact with the air. Water was supplied at about 60° F. from the town's main through the annular space between the tubes, and the inlet and outlet temperatures were measured by mercury thermometers inserted in deep pockets. The weight of water passing through was obtained by tanks on weighing machines, and the heat taken up by the water could thus be obtained. This was equated to the heat lost by the air, neglecting external losses, and using 0·24 as the specific heat of the air the weight of air-flow was thus calculated. All the parts of the apparatus were well insulated by asbestos cement and felt covering. An orifice outlet was arranged as shown dotted in Fig. 77 by which it was hoped to get an estimate of the air-flow, but this was not successful, owing to the velocity of approach to the orifice being excessive, and its value unknown. This orifice was afterwards discarded, and replaced by a divergent mouthpiece having several modifications to be discussed later. In designing the apparatus the author chose to let the air flow through the inner tube and the water outside, for the particular reasons that the external losses of heat would be reduced to a minimum, and that it was hoped to convert some of the kinetic energy of the leaving air into the form of pressure energy, whereas, had the water been sent through the tube, and the air through the annular space, no recovery of the kinetic energy of the leaving air could be expected. This feature is discussed more fully on p. 182.

The principal results of these experiments are shown in Figs. 78 to 82. Fig. 78 shows how the rate of heat transmission between the air and water varies with the weight of air-

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* Royds and Campbell, "The Possibilities of Flue Gas Economisers on Board Ship," *Trans. Inst. Engs. and Shipbds. in Scotland,* Vol. LV, 1911–12.

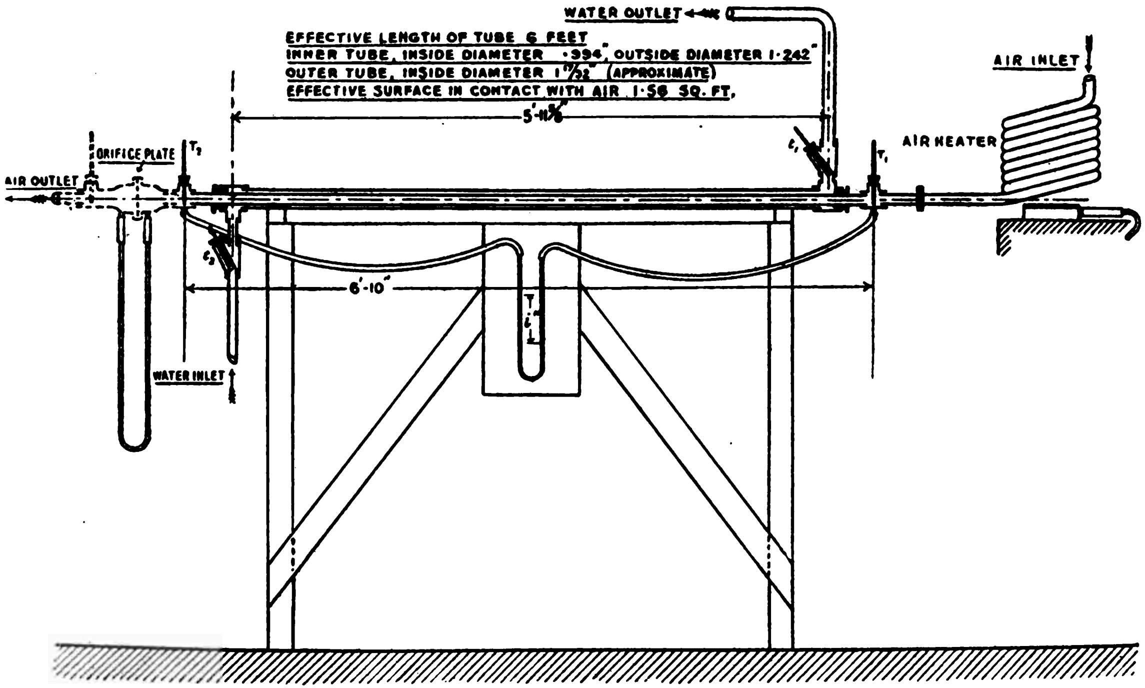
WATER OUTLET
EFFECTIVE LENGTH OF TUBE 6 FEET
INNER TUBE, INSIDE DIAMETER ·394″, OUTSIDE DIAMETER 1·242″
OUTER TUBE, INSIDE DIAMETER 1 17/32″ (APPROXIMATE)
EFFECTIVE SURFACE IN CONTACT WITH AIR 1·56 SQ. FT.
5′-11⅝″
AIR INLET
AIR HEATER
t_1
T_1
T_2
ORIFICE PLATE
AIR OUTLET
t_2
6′-10″
WATER INLET
T
t

Fig. 77.—Apparatus used in Royds and Campbell's experiments.

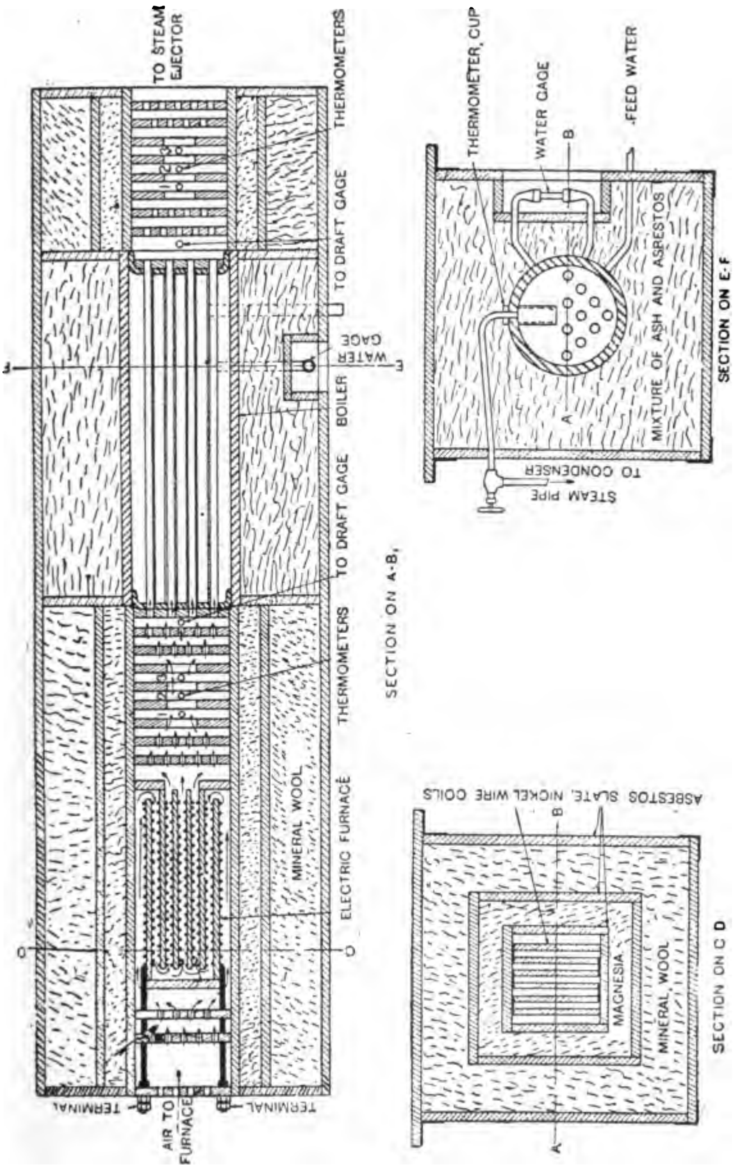

FIG. 68.—Small boilers used in Kreisinger and Ray's experiments.

The Boilers.—Experiments were made with four horizontal boilers having the dimensions given in Table 18.

TABLE 18

Number of boiler	1	2	3	4
Length of boiler, outside to outside of boiler ends, inches l	8·28	8·28	16·125	8·28
Number of copper tubes	10	10	10	10
Actual outside diameter of tubes, inches	·252	·313	·252	·3745
Actual inside diameter of tubes, inches	·175	·230	·175	·2928
Total area of cross section of tubes, square feet $= a_1$	·00167	·002885	·00167	·00467
Total heating surface (gas side) of tubes, square feet	·3162	·4155	·6156	·529
$\frac{l}{m_1}$ for heat-flow	189	144	369	113
$\frac{l}{m_1}$ for resistance to air-flow	200·5	153	380	120

The boilers were constructed of a piece of standard 4-in. wrought-iron pipe for the outside shell of each boiler. The boiler ends were made of $\frac{1}{8}$-in. sheet copper and were riveted to the 4-in. pipe. All the boilers had ten tubes of thin copper, which were expanded and soldered into holes in the boiler ends, the positions being indicated by the sections in Fig. 68. Both boiler ends were covered with $\frac{1}{4}$-in. asbestos paper and slate so that only the heating surface of the tubes was really effective. This is the reason for the comparatively large difference in the values of $\frac{l}{m_1}$ for heat-flow and for resistance to the air-flow shown in the above table. Each boiler was fitted with a water-gauge glass, a thermometer cup for measuring the temperature of the steam, a pipe for feeding the boiler, and a pipe for letting out the steam. The boiler was fed with distilled water obtained by condensing steam from the main boiler of the plant.

The electric furnace is illustrated in two sectional views in Fig. 68. It consisted of six coils of pure nickel wire wound around rectangular pieces of asbestos slate, $\frac{9}{32}$ in. thick, $3\frac{7}{8}$ in. wide, and 12 in. long. The nickel wire was ·072 in. diameter, about No. 13 B. and S. gauge, there being 51 turns in each coil, so that the total length of the wire in the furnace was 204 ft. In most of the tests the coils were connected in series; with high temperatures and high pressure drops or "draughts" three coils were connected in series, and the two sets were connected in parallel. A water rheostat was used for adjusting the electrical energy to each temperature and draught. The coils were placed vertically in the front portion of the asbestos slate box with a space of about $\frac{5}{16}$ in. between them, and were so baffled that the air passed along the coils three times before it left the furnace. Direct current of 220 volts was used and the energy was measured by an integrating wattmeter.

With temperatures up to about 900° F. there was no particular trouble, but when the temperature of the air was raised to 1200° F. and 1500° F. the nickel wire seemed to crystallise and become very brittle after cooling. The coils usually failed by fusing after they had been in use ten hours a day for six to ten days. It must be understood that in order to raise the temperature of the air to 1500° F. the coils were sometimes heated close to the point of fusion (2600° F.).

The space between the furnace and the boiler was fitted with eight perforated screens, and the four middle screens had in their centres round 1¼-in. holes. In the spaces between these four screens three thermometers were so inserted that their bulbs were in or near the centre line of the 1¼-in. holes, and an inspection of Fig. 68 enables the arrangement to be understood. The object of these screens was to confine the air around the bulbs of the thermometers in order that its temperature could be more accurately measured. The two screens nearest the furnace and the two nearest the boiler each had thirty-two ¼-in. holes, so drilled that the thermometers were perfectly screened from direct radiation from either the boiler or the furnace. Thus the enclosure in which the thermometers were situated had all the surfaces at practically the same temperature as the thermometers, and therefore the bulbs should have the same temperature as the air.* A similar arrangement was employed for obtaining the temperature of the air leaving the boiler. Tubes for measuring the pressure of the air were inserted on both sides of the boiler between the boiler end and the first screen.

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* Refer to p. 18, for a discussion as to the importance of this matter at high temperatures.

The asbestos-slate box forming the body of the apparatus was 1 ft. high by 1 ft. wide by 5 ft. long. The spaces enclosing the furnace and behind the boiler were 4 in. square, so that the total thickness of the insulating walls was 4 in. The walls around the furnace consisted of a ¼-in. lining of asbestos slate, and a 1-in. layer of magnesia boiler covering. The space between the layer of magnesia and the outside sheet of the box was filled with mineral wool. The space around the boiler was filled with a mixture of wood ashes and asbestos.

The temperatures up to 700° F. were measured with mercurial thermometers, which were compared with a standard thermometer calibrated by the Bureau of Standards in Washington. Temperatures above 700° F. were measured by calibrated platinum, platinum-rhodium thermo-couples directly exposed to the hot air.

Although great care was taken in placing and in screening the thermo-couples, as was described on p. 161, considerable difficulty was experienced in measuring the initial and final temperatures of the air. With the initial temperatures of 1200° F. and 1500° F., the three thermo-couples would at times read as much as 100° F. apart, and for no apparent reason. Generally the thermo-couple closest to the furnace read higher than the other two. The only reasonable explanation of this fact was that in spite of the screens this thermo-couple received some heat by radiation from the furnace, whereas the one nearest the boiler lost some heat by radiation to the boiler. The thermometers on the side of the leaving gas read from 10° F. to 20° F. apart. It was difficult to come to any conclusion as to which of the thermometers or thermo-couples were wrong, and it was thought best to take the average of the three readings at either end. These results illustrate the great difficulties experienced in attempting to measure accurately high temperatures of a moving gas.

Before starting a test the apparatus was brought to the temperature at which the test was to be run, and after a test was started all conditions were kept as nearly uniform as possible. The duration of each test was from thirty minutes to two hours. The higher the draught and the temperature the shorter was the duration of the test made.

In all but a few of the experiments the steam was generated at atmospheric pressure, the steam temperature being in the neighbourhood of 212° F., and the feed water temperature varied in different tests from about 62° F. to about 144° F., but generally it was in the neighbourhood 70° F. to 100° F.

The weight of air w_1 passing through the boiler in pounds per second was calculated from the weight of steam condensed and the heat given per pound of steam. It was taken that the steam was without moisture as it left the boiler.

If W=weight of steam condensed, lbs. per sec.
t_s=temperature of steam, °F.
t_w=temperature of feed water, °F.
T_1=air temperature at inlet, °F.
T_2= ,, ,, ,, outlet, °F.

Then w_1 was estimated as follows :—

$$W\{965{\cdot}7+(t_s-t_w)\}={\cdot}2375w_1(T_1-T_2)$$

$$\text{or, } w_1=\frac{W\{965{\cdot}7+(t_s-t_w)\}}{{\cdot}2375(T_1-T_2)}$$

Altogether the results of 237 tests are tabulated in the bulletin, and are too numerous to reproduce here in tabular form. Table 19, however, indicates the conditions of the experiments sufficiently well for the present purpose. The values in column 3 represent the average air inlet temperatures during each of the series of tests. In columns 4 and 5 are given respectively the maximum and minimum mean air inlet temperatures in each series, each value, of course, representing the average reading of the three thermometers in the particular tests. Similarly, columns 6 and 7 contain respectively the maximum and minimum mean air outlet temperatures. In each of the series the principal independent variable condition was the draught and the consequent variation of the rate of flow of the air through the tubes. This is subsequently indicated in the various graphs, so that a tabulation of the rate of air-flow is hardly necessary.

TABLE 19

Boiler Number.	No. of tests in series.	Air inlet temperatures.			Air outlet temperatures.		Steam temperatures °F.	Draught, inches water.		Coefficient * of tube resistance *f*.
		Average in each series. °F.	Maximum in each series. °F.	Minimum in each series. °F.	Maximum in each series. °F.	Minimum in each series. °F.		High-st in the series.	Lowest in the series.	
1	2	3	4	5	6	7	8	9	10	11
1.	19	497	504	489	268	223	209 to 211	16·9	·3	·019
	19	695	702	677	320	266	209 to 211	13·8	·5	·0142
	12	982	998	965	379	292	210	14·0	·5	·009
2.	27	495	511	481	313	243	210 and 211	22·9	·1	·0396
	16	675	684	658	378	295	210 to 212	24·0	·2	·0239
	29	900	957	860	482	324	mostly 211 to 216 a few 242 to 277	15·1	·1	·0154
	28	1217	1255	1176	626	374	211 to 227	23·6	·1	·0099
	11	1519	1540	1492	696	454	211 to 223	7·0	·3	·0069
3.	19	684	688	677	289	231	210 to 216	19·7	·5	·0088
	14	884	912	867	312	232	211 to 214	21·3	1·0	·0088
	10	1212	1232	1195	370	268	211 to 218	20·5	·85	·0071
	11	1511	1542	1485	404	245	211 to 222	20·1	·5	·006
4.	10	691	697	680	434	391	211 to 214	14·1	·5	·021
	9	898	913	880	509	434	211 to 216	13·2	·5	·0253
	3	1195	1207	1180	602	529	211 to 213	1·2	·5	·0153

* Calculated as on p. 169, after allowing for the pressure drop required to create the kinetic energy of flow at inlet to the tubes but neglecting any losses at inlet or recovery at outlet.

It would be evident from the description of the apparatus and of the conditions of the tests that great precautions were taken by the experimenters to ensure a reasonable degree of accuracy in their measurements.

The author has calculated, from the experimental values tabulated in the bulletin, the rate of heat transmission h in B.Th.U's. per square foot (air side) per second per deg. Fahr. difference between the air and the water, and the respective values of

$$\frac{w_1}{a_1}=\frac{\text{air-flow, lbs. per sec.}}{\text{area of flow, sq. ft.}}$$

The mean difference of temperature was given by

$$\frac{T_1-T_2}{\log_e\left(\frac{T_1-t}{T_2-t}\right)}$$

where t was taken to be a few degrees below the steam temperature, to allow for the low feed-temperature. The rates of heat transmission h were plotted on the base $\frac{w_1}{a_1}$ for each boiler, and Fig. 69 has been reproduced for boiler No. 2 to show the character of the results obtained. The whole of the mean lines, however, were reproduced in Fig. 70, without showing the plotted points, and these indicated the straight line relation connecting the rate of heat transmission h with $\frac{w_1}{a_1}$ within the limits of the experiments. The average inlet temperatures of the air were indicated on the lines, and it would be noticed that the temperature of the air had but little influence upon h. For some reason the values for boiler No. 3, which had tubes nearly twice the length of those in the other three boilers, were substantially lower than the others. An inspection of the results for boilers Nos. 1, 2, and 4, would also indicate that the rate of heat transmission h increased as the diameter of the tube decreased.

The calculated tube efficiencies

$$\frac{T_1-T_2}{T_1-t}$$

for boiler No. 2 are also shown plotted in Fig. 69.

To analyse further the results of these experiments the author calculated the various values of M and c_1 by means of

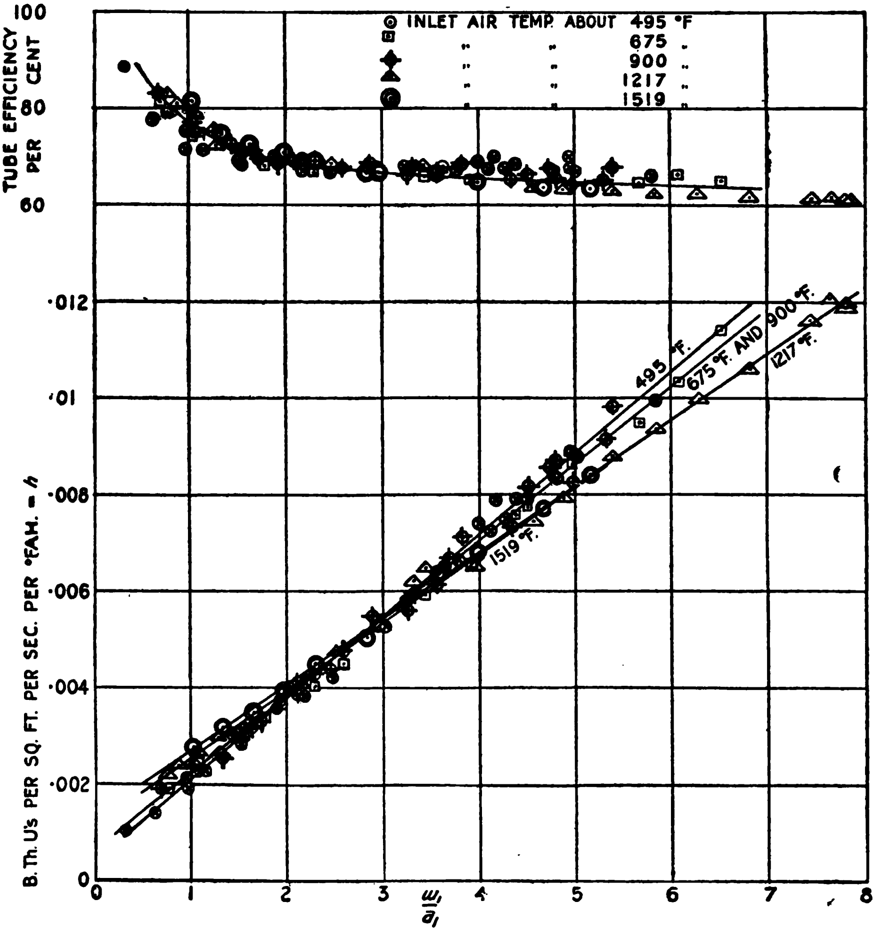

FIG. 69.—Rate of heat transmission for boiler No. 2, from Kreisinger and Ray's experiments.

the equation 23, p. 149, and using the value for $c_2\frac{w_2}{a_2}$[*] $=C_2=\cdot 03$ for boiler conditions on the water side of the tubes. The results obtained are indicated in Fig. 71 by the various mean curves, and as the actual plotted points were found to lie fairly close to the respective mean lines, it was not thought to be worth while to reproduce the points.

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[*] The conditions of circulation when water is boiling are not the same as when water is being merely heated and rising in temperature. Provided that large bubbles of steam are not allowed to cling to the heating surfaces due to

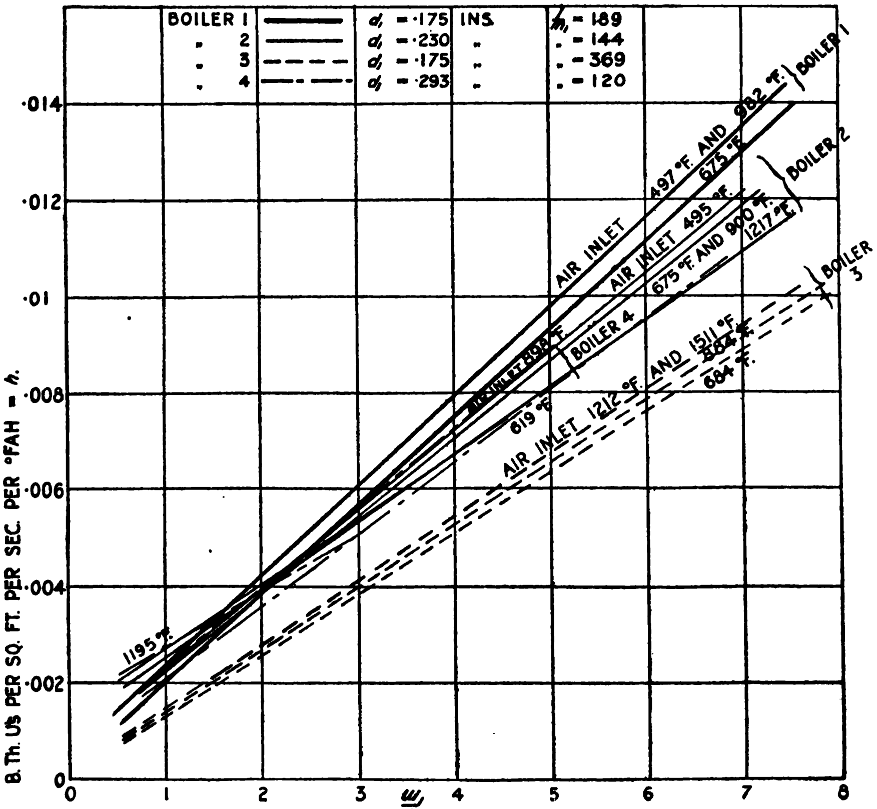


FIG. 70.—Rates of heat transmission from Kreisinger and Ray's small boiler experiments.

On referring to Fig. 68, p. 159, it would be seen that draught connections were made at both the inlet to and outlet from the

bad circulation of the water, it is probable that the improvement of circulation due to the presence of steam bubbles lowers the resistance to the heat-flow on the water side. The value of $c_2\frac{w_2}{a_2}=C_2=\cdot 03$ instead of $C_2=\cdot 0131$, given on p. 180 is intended to allow for this reduced resistance.

boiler tubes, which gave the pressure drop through the tubes. This pressure drop included not only the resistance of the tubes, but also the end resistances. At the inlet end, for instance, a certain pressure drop was necessary to create the kinetic energy of flow into the tubes, and besides this there would be some loss in converting the pressure energy into kinetic energy. At the outlet end, however, there may have

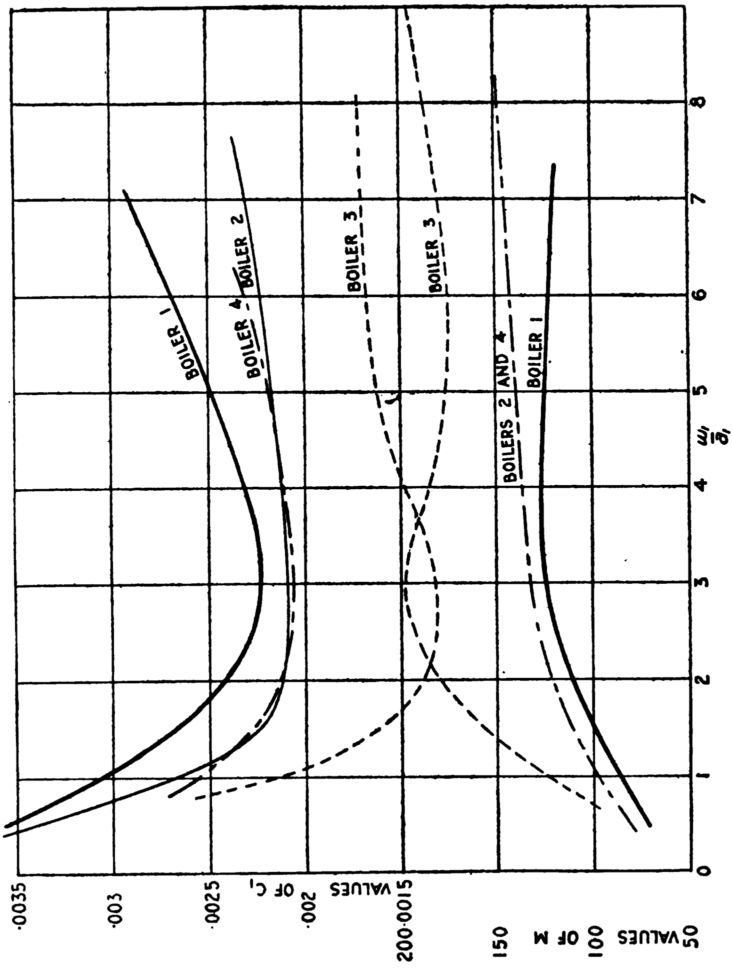


FIG. 71.—Values of "M" and "c_1," from Kreisinger and Ray's small boiler experiments.

been a conversion of a small portion of the kinetic energy at exit into pressure energy. In the following calculations any loss of energy at the inlet to the tubes and any recovery of energy at the outlet have been neglected.

If $\triangle P$=total drop of pressure, lbs. per sq. ft.
δP=pressure drop due to tube resistance, lbs. per sq. ft.
v_1=air velocity at inlet, ft. per second.
ρ_1=air density at inlet, lbs. cub. ft.

Then, for approximate calculation, it might be taken that,

$$\triangle P=\delta P+\frac{v_1^2\rho_1}{2\times 32{\cdot}2}$$

or, since, $v_1\rho_1=\frac{w_1}{a_1}$

$$\triangle P=\delta P+\frac{v_1}{64{\cdot}4}\times\frac{w_1}{a_1}$$

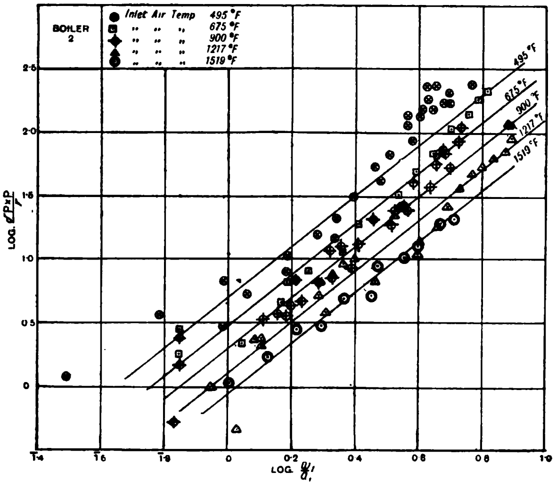


FIG. 72.—Relation between flow of gas through a tube and the resistance to flow from Kreisinger and Ray's experiments.

From the tabulated data the writer has calculated the various values of δP. Making use of equation 23, p. 85, values of $\left(\frac{\delta P\times P}{T}\right)$ were then calculated, and log $\left(\frac{\delta P\times P}{T}\right)$

differences in the arrangement and positions of the manometer connections at the ends of the tube, the one arrangement including some end resistances and the other measuring only the resistance of the tube itself. In a general way, then,

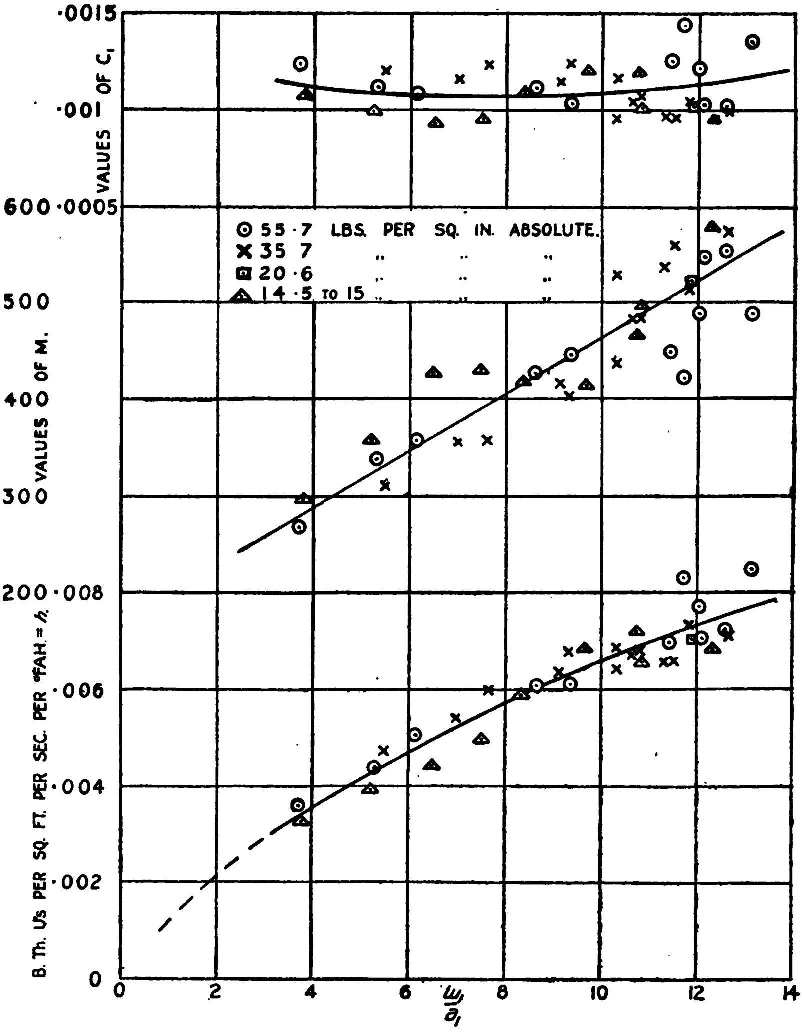
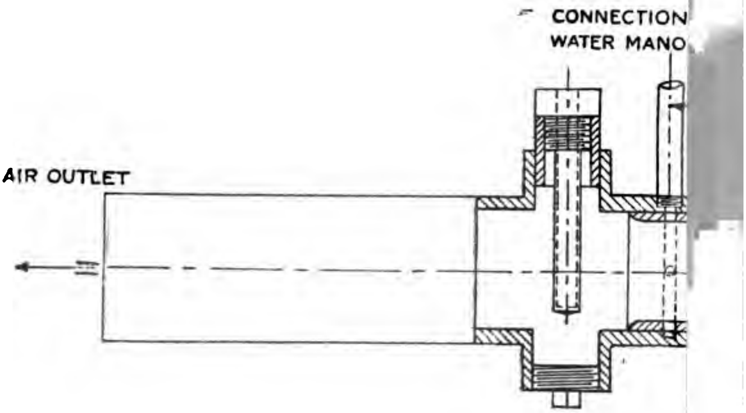
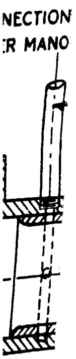


FIG. 88.—Rates of heat transmission at various pressures of the air, from Royds' experiments.


CONNECTION
WATER MANO
AIR OUTLET

these experiments confirm the Reynolds law of convective heat transmission within the limits of the experiments, and also show that the coefficient of resistance f is practically independent of the gas pressure.



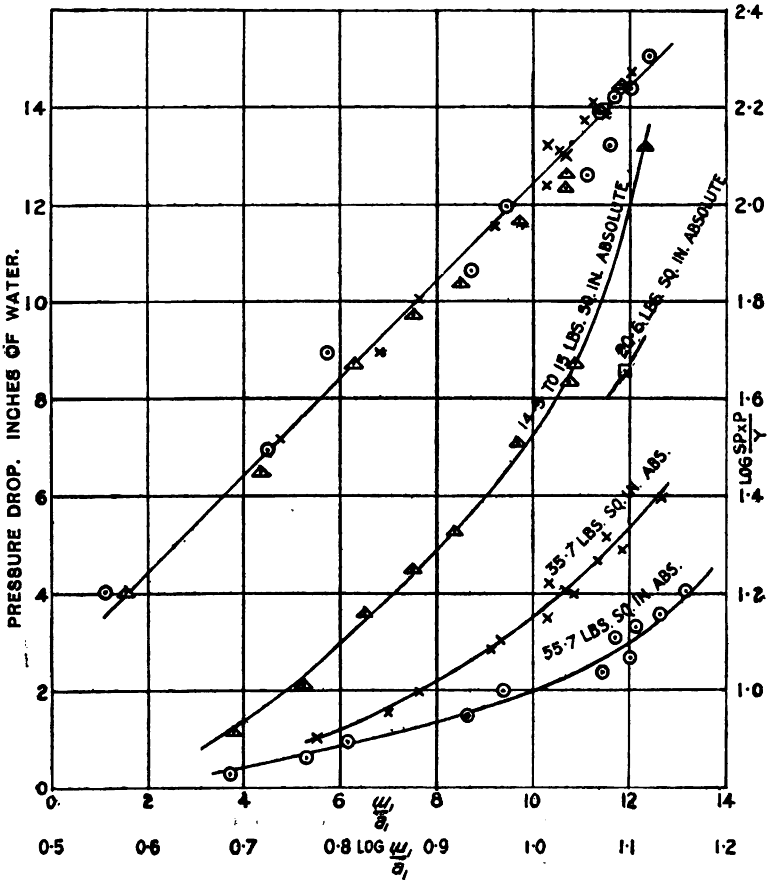
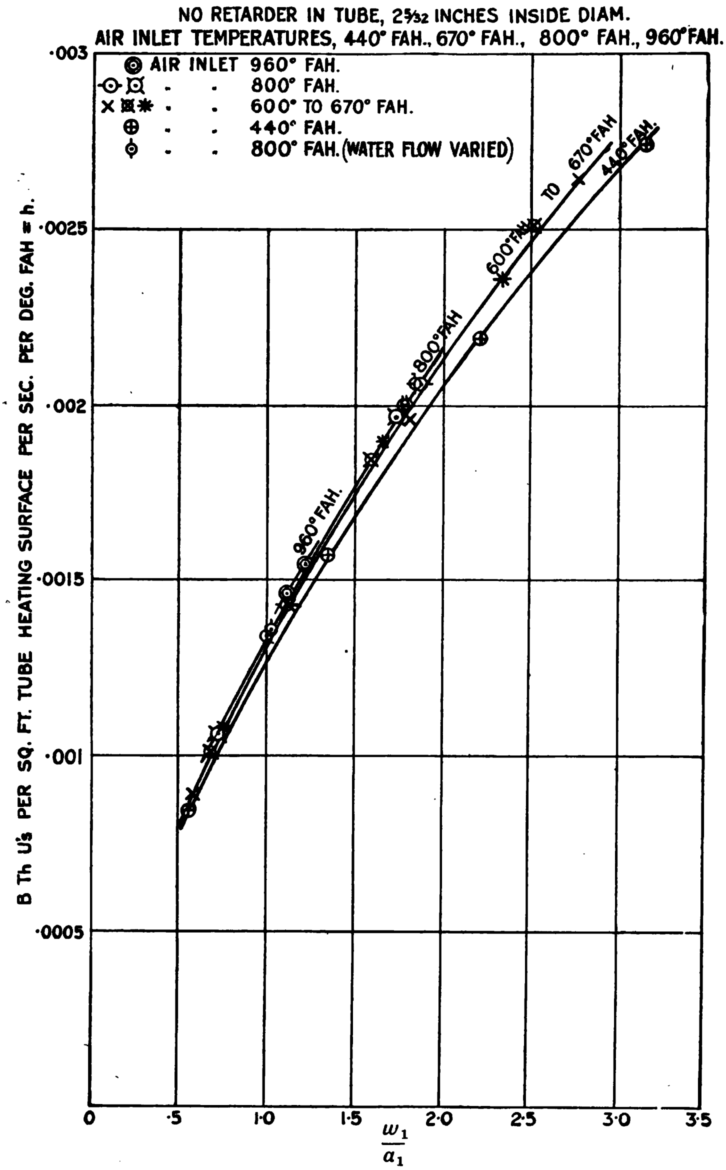


FIG. 89.—Resistance to the flow of the air in Royds' experiments at different air pressures.

Another apparatus was arranged and constructed in a similar manner to the one just described but having a lap-welded boiler tube of $2\frac{1}{2}$ in. outside and $2\frac{5}{32}$ in. inside diameter, enclosed in another tube of $2\frac{27}{32}$ in. inside diameter.

The detailed construction of this apparatus is shown in Fig. 90, and when in use it was well covered with 80% magnesia. The air from the heated coil was allowed to flow through the tube, with town's water flowing in the counter-current direction on the outside. The inlet temperature of the air was measured by a calibrated platinum resistance thermometer having a porcelain cover in direct contact with the air. The other temperatures were obtained by calibrated mercury thermometers inserted in deep pockets. The inlet temperature of the water was about 60° F., and the weight of water was obtained in tanks on weighing machines. A large number of tests were made by the author with air inlet temperatures varying from about 430° F. to over 1000° F., but for several weeks after being put into use the rate of heat transmission, *h*, decreased slightly but gradually. Eventually it settled down to consistent values and has remained at these values since. The final character of the values of *h* is shown in Fig. 91, plotted on the base $\frac{w_1}{a_1}$, with air inlet temperatures varying from about 440° F. to about 960° F. Several sets of tests were also made with the air-flow constant, having an inlet temperature 800° F., but varying the water velocity from ·145 to ·58 ft. per second. The results obtained under these conditions are also plotted in Fig. 91, and showed that the quantity of water flowing had but little influence on the value of *h*. It is seen again that the temperature of the air has an influence on the value of *h*. Values of M and c_1 have been calculated for these experiments from equations 2 and 1, p. 179, and these have been plotted in Fig. 92. The resistance offered to the flow of the air through the tube is shown in Fig. 95, p. 198.

The Influence of Retarders in Tubes.—To increase the rate of heat transmission through boiler tubes, twisted strips of metal, generally called "retarders," are sometimes inserted. It seemed to be a prevailing idea that the spiral retarder increases the heat transmission because of the spiral motion given to the gas flowing through the tube. To test this, and to ascertain to what extent the retarder influences the rate of heat transmission, experiments were made on the apparatus



$\frac{w_1}{a_1}$

FIG 91.—Rate of heat transmission from air to water, from Royds' experiments.

shown in Fig. 86, p. 186, with retarders ·95 in. wide and ·05 in. thick inserted in the 1-in. tube, and having 0, 2, 4, and 6 complete turns or spirals in a length of 6·7 ft. Experiments were thus made with air-inlet temperatures from 360° F. to 630° F., and the results obtained showed that the retarders had a considerable influence on the heat transmission, and also showed that the number of turns had hardly any influence up to four turns in the length of 6·7 ft., but beyond this the number of turns caused an increase in the rate of heat trans-

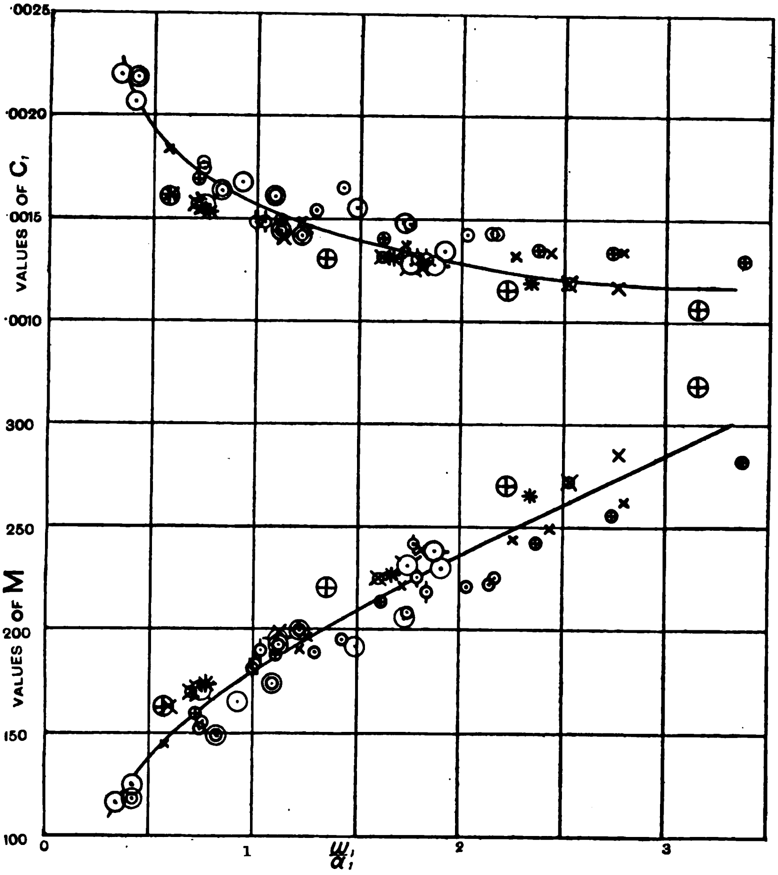


FIG. 92.—Values of "M" and "c_1," from Royds' experiments.

mission. The straight retarder was tried placed in the horizontal plane as well as in the vertical plane. No appreciable differences in the results were observed. The detailed results are not reproduced here because a similar but more extensive series of experiments were made with retarders inserted in the tube of $2\frac{5}{32}$-in. bore (Fig. 90). Altogether nine retarders were used in this latter-mentioned apparatus, each $1\frac{15}{16}$ in. wide and ·1 in. thick, and having respectively one complete turn in the lengths ∞, 89, 44·5, 29·7, 22·2, 16·8, 15, 13·1, and 10·9 in., numbered 0 to 8 respectively.

A large number of tests were made at various air-inlet temperatures from about 430° F. to over 1000° F., with these retarders used in the manufactured condition. The results obtained again showed that up to a critical point of one turn in about 20 in., the number of turns has very little influence on the heat transmission. These results, however, were not altogether consistent among themselves, and this was ascribed to variations in the surface condition of the retarders, some of which were covered with small patches of rust. It was decided, therefore, to repeat some of these experiments with some of the retarders blackleaded, and afterwards when sooted with lampblack. The rates of heat transmission in both cases were higher than with the retarder surfaces in the manufactured condition, but were now quite consistent, and, when blackleaded, were generally slightly lower than when sooted. It is proposed, therefore, to discuss further only the data obtained with the sooted retarders, as this most nearly represents boiler conditions as regards the surface of the retarders.

Fig. 93 shows the results obtained for the straight retarder 0 in the sooted condition, with air-inlet temperatures 630° F., 800° F., and 1000° F. respectively. Similar data were obtained with the sooted retarders 2, 4, 6, and 8, but as it is hardly possible to show all the various values, Fig. 94 has been drawn to illustrate the relation between the rate of heat transmission, the number of turns in the retarder, and $\frac{w_1}{a_1}$, with the air-inlet temperature 800° F. The plotted points have not been reproduced, but in all cases the curves represent the experimental results quite as well as those in Fig. 93. It would be seen that

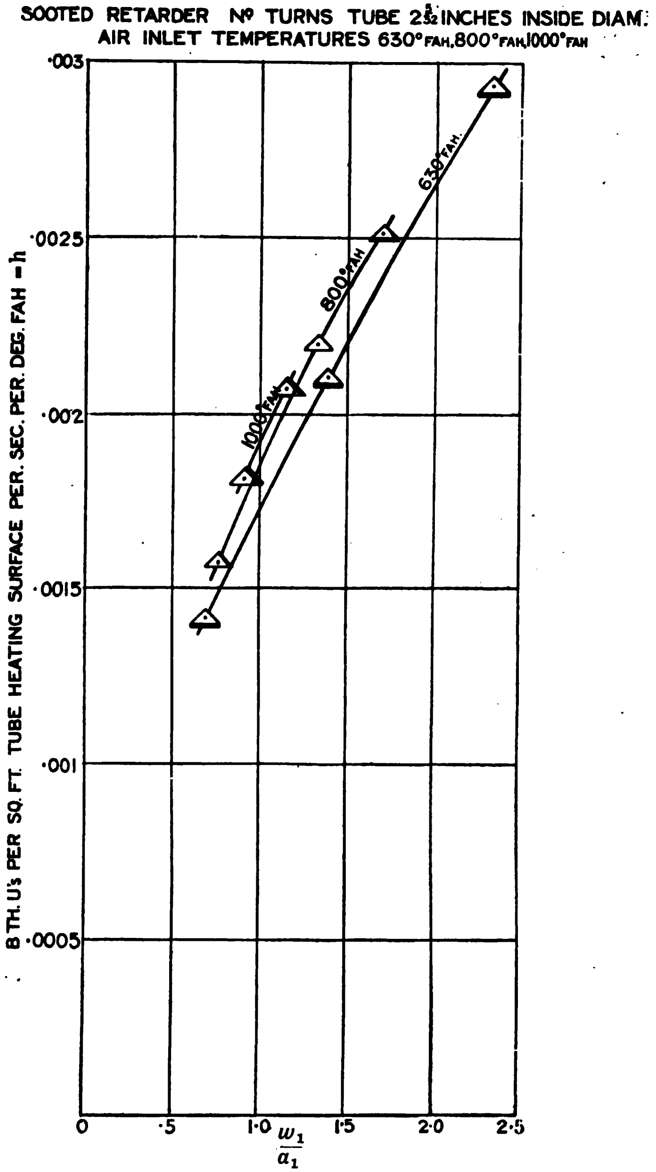


FIG. 93.—Rate of heat transmission from air to water with straight retarder having sooted surfaces inserted into tube, from Royds' experiments.

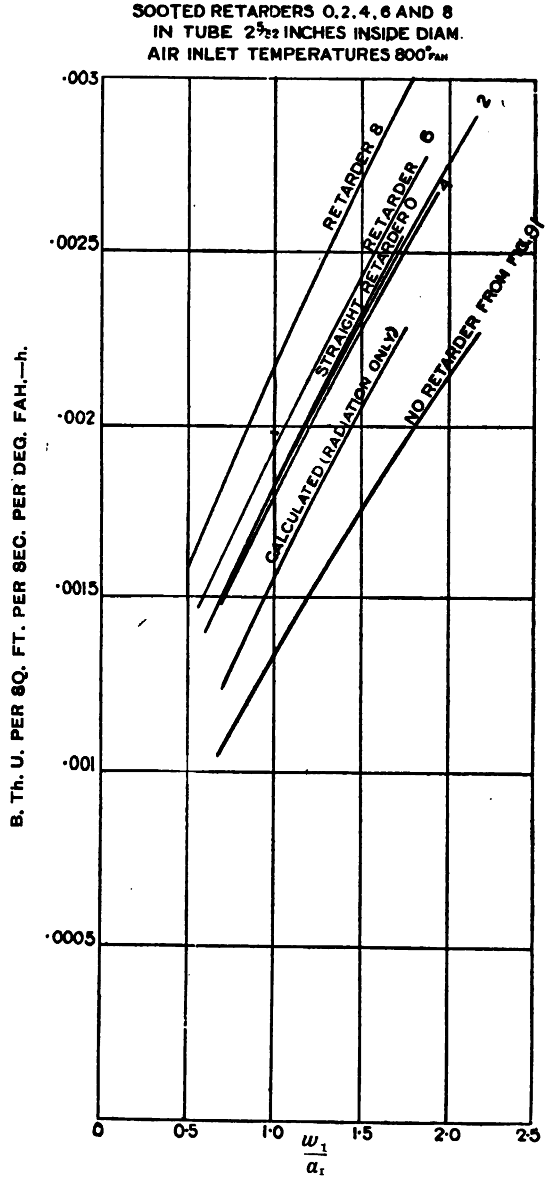


Fig. 94.—Rate of heat transmission with twisted retarders, from Royds' experiments.

the retarders 0, 2, and 4 gave practically identical values of the rate of heat transmission, but retarders 6 and 8 showed a progressive increase. This was also found to be the case with the other air-inlet temperatures, thus showing that there is a critical number of turns in the neighbourhood of one turn in 20 in.; that is, from the straight retarder up to one turn in about 20 in. the number of turns has little or no influence on the rate of heat transmission.

For purposes of comparison the curve in Fig. 94, representing the results for the plain tube without retarder with the air-inlet temperature of 800° F., is reproduced from Fig. 91, and the difference between this curve and the others shows the influence of the retarders on the rate of heat transmission. Some of this influence is no doubt due to radiation from the surfaces of the retarders. The effect of radiation can be estimated approximately in the following manner :—

Let T = gas temperature, °F.
t = water temperature, °F.
T_r = retarder temperature, °F.
h = rate of heat transmission, gas to water.
q = rate of heat transmission, gas to retarder.
h' = rate of heat transmission to water (including radiation from retarder).
d_1 = diameter of tube.
d = width of retarder.

Then, in a length δx of the tube, it might be stated that,

(Heat absorbed by tube) = (Heat given to tube by the gas) + (Heat radiated by retarder.)

But the heat radiated by the retarder would be equal to that which it absorbed from the flowing gas, which means,

$$\text{Heat radiated by retarder} = q\,(T - T_r)\,2\,d\,\delta x \quad . \quad . \quad . \quad . \quad (1)$$

$$\text{Thus, } h'\,(T-t)\,\pi d_1 \delta x = h\,(T-t)\,\pi d_1 \delta x + q\,(T-T_r) 2 d \delta x \quad . \quad . \quad (2)$$

If h and q are equal and the same as the value given by the plain tube without retarder, an approximate estimate of h' can be made for each end of the tube and the average value obtained. For this purpose it may be taken that the sooted retarders radiate practically as a black body, and thus,

$$q\,(\mathrm{T}-\mathrm{T}_r)\,2d\,\delta x=\frac{1600}{3600}\left\{\left(\frac{\mathrm{T}_r+460}{1000}\right)^4-\left(\frac{t+460}{1000}\right)^4\right\}2d\,\delta x \quad . \; . \quad (3)$$

With given data the value of T_r can be found from this equation (3), and using this value in equation (2) the corresponding values of h' are obtained. For example, in one test with the $2\frac{5}{32}$-in. bore tube, using the straight retarder 0,

$$\mathrm{T}_1=815^\circ\ \mathrm{F.},\ \mathrm{T}_2=319^\circ\ \mathrm{F.},\ t_1=87{\cdot}9^\circ\ \mathrm{F.},\ t_2=62{\cdot}0^\circ\ \mathrm{F.},$$

$$\frac{w_1}{a_1}=1{\cdot}33\ ;\ \text{and } q=h={\cdot}00163 \text{ from Fig. 91.}$$

The calculated values of T_r by equation (3) are,

$\mathrm{T}_r=553^\circ$ F. at inlet, and 264° F. at outlet,

and from equation (2),

$h'={\cdot}00189$ at inlet, and ·00176 at outlet, giving a mean of about ·00182.

In this manner a series of values for h' were calculated, and the curve marked "calculated" in Fig. 94 represents the results for an air-inlet temperature of 800° F. It would be noted that these calculated values fell far short of the experimental values for the sooted retarders. For purposes of reference, comparisons have been made in Table 22 between the experimental and calculated values of h' for the straight retarder 0 and for retarder 8. The experimental values of h' for retarders 0 and 8 are given in columns 3 and 10 respectively; column 4 shows the values for no retarder obtained from Fig. 91, and column 5 represents the calculated values when the effect of radiation only is added to the no retarder values. Column 7 shows the difference between columns 3 and 4, as a percentage of the values in column 4, and similarly column 9 shows the difference between columns 3 and 5, as a percentage of column 4. Similar calculations made for retarder 8 are given in columns 10 to 14.

The resistance offered to the flow of the air through the $2\frac{5}{32}$-in. bore tube was measured by making connections between two points of the tube, as shown in Fig. 90, with the two legs of an inclined U water gauge. To illustrate the character of the results obtained, Fig. 95 is reproduced. The resistance for a length of 7·17 ft., measured in inches of water,

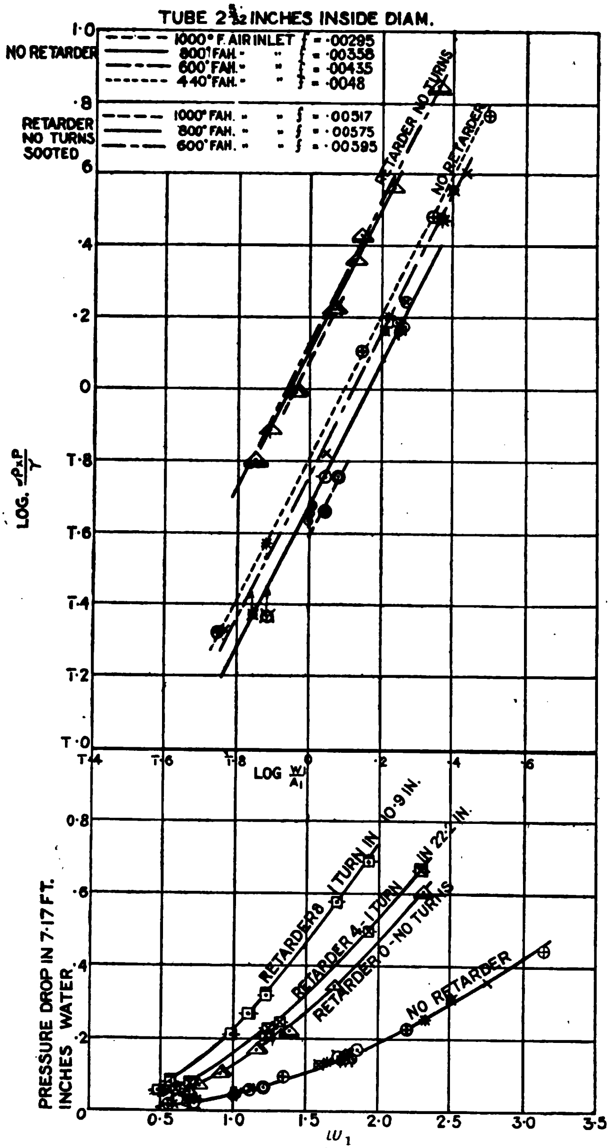


FIG. 95.—Resistance to the flow of air through a tube, both with and without retarders, from Royds' experiments.

TABLE 22

Air inlet temp. °Fah.	$\frac{w_1}{a_1}$	Experimental values from Fig. 93 for retarder O.	Corresponding values from Fig. 91 for no retarder.	Calculated mean values on adding radiation to Col. 4.	Col. 3—Col. 4.		Col. 3—Col. 5.		Experimental values for retarder 8.	Col. 10—Col. 4.		Col. 10—Col. 5.	
					Actual.	Percentage of Col. 4.	Actual	Percentage of Col. 4.		Actual.	Percentage of Col. 4.	Actual.	Percentage of Col. 4.
1000	·905	·00180	·001275	·00152	·000525	41·1	·00028	22·0	·00215	·000875	69	·00063	49·5
	1·155	·00208	·00149	·00175	·00059	39·6	·00033	22·2	·00243	·00094	63	·00068	45·6
800	·772	·001575	·00111	·00132	·000465	41·9	·000255	23·0	·00191	·00080	72	·00059	53·2
	1·33	·00220	·00163	·00182	·00057	35·0	·00038	23·3	·00254	·00091	56	·00072	44·2
	1·705	·00252	·001935	·00224	·000585	30·2	·00028	14·5	·00292	·000985	51	·00068	35·2
630	·706	·00142	·00100	·00124	·00042	42·0	·00018	18·0	·00175	·00075	75	·00051	51·0
	1·39	·00211	·00166	·00190	·00045	27·1	·00021	12·7	·00248	·00082	49·5	·00058	35·0
	2·325	·00293	·00236	·00264	·00057	24·2	·00029	12·3	—	—	—	—	—
1	2	3	4	5	6	7	8	9	10	11	12	13	14

is there shown plotted on the base $\frac{w_1}{a_1}$ for no retarder, and for retarders 0, 4, and 8. All the curves show that the resistance for the length of 7·17 ft. of the tube is not affected by the temperature of the flowing air. It would be observed also that the resistance for retarders 0 and 4 was about twice that for the plain tube, while retarder 8 shows a large increase of resistance over retarders 0 and 4. On the whole it was found that the number of turns in the retarder, from the straight retarder 0 to retarder 4, did not greatly affect the resistance, and, therefore, it may be said that the number of turns has practically much the same effect on the resistance as on the rate of heat transmission. The basis of the Reynolds law of heat transmission would lead one to expect some such result.

To a first approximation the resistance to the flow of fluids is commonly represented by equation (23), p. 85. Accepting this equation the values of log $\left(\frac{\delta P \times P}{\tau}\right)$ have been plotted in Fig. 95 on the base log $\left(\frac{w_1}{a_1}\right)$ for the $2\frac{5}{32}$-in. tube without retarder, and for the straight retarder 0, the straight lines being at a gradient of nearly 2. The values of f obtained are also tabulated in Fig. 95, and it would be seen that these values decrease as the air-inlet temperature increases. This was to be expected because of the changing temperature of the air flowing through the tube, and the consequent changes of momentum.

The principal advantage, then, of using retarders in boiler tubes as compared with plain tubes is the reduction of length of the tubes and of the boiler, or an increase of efficiency. This is obtained, however, at the expense of increased resistance to the flow of the gases. Apart from these considerations it is probable that, for a specified resistance to the flow, the plain tubes are quite as effective as with retarders inserted. For example, referring to Fig. 95, p. 198, it is seen that with $\frac{w_1}{a_1}=1$ the resistance for the 7·17 ft. length of tube with retarder 4 inserted was about ·16 in. of water. For the same resistance without retarder (neglecting the pressure required to create the extra kinetic energy of flow) the corresponding value of $\frac{w_1}{a_1}$ was about 1·8. A reference to Fig. 94, p. 195, shows that with $\frac{w_1}{a_1}=1$ the rate of heat transmission with retarder 4 was ·0018, whilst without retarder at $\frac{w_1}{a_1}=1 \cdot 8$ the rate was ·002.

In ordinary boilers all the tubes are arranged in parallel, that is, there is only one pass, but the above results indicate that whenever multiple passes can be incorporated in a design by which means the length of the path of the gases can be altered in the design without increasing the overall length of the apparatus, to introduce retarders in the tubes would be a doubtful advantage.

The Influence of Specific Heat of Fluids on the Rate of Heat Transmission.—Reverting to equation 11, p. 119, which expresses the Reynolds law of heat transmission between a fluid and a surface, it is seen that the rate of heat transmission, h, according to this law should increase with the specific heat of the fluid. This was shown to be the case in Nusselt's experiments on p. 127, and is further confirmed by comparing the values of h for a gas and for water at comparable values of $\frac{w}{a}$. For example, it would be seen from Fig. 66 that with the air-flow $\frac{w_1}{a_1}=120$ the value of h_1 would be about ·09. At the value of $\frac{w_2}{a_2}=120$ for water an average value of h_2 from Fig. 54 would be about ·33, which is about four times the value for air, and this is about the ratio of the specific heats of water and air. Similarly, comparing the value of h_1 from Fig. 50 when the air-flow $\frac{w_1}{a_1}=20$ with the value of h_2 from Fig. 54 when the water-flow $\frac{w_2}{a_2}=20$, it would be seen that these values are respectively ·016 and ·07. The latter value is again roughly four times the former.

In some of the experiments which have been discussed it was noted that the temperature of the gas has an influence on the rate of heat transmission. This is no doubt due to a combination of circumstances, each one circumstance perhaps of secondary importance, but all of which influence the result in the same direction. It is well known that the specific heat and the conductivity of a gas increase with the temperature and theory suggests that both of these factors affect the rate of heat transmission. Also there would be a small amount of radiation from the gas, particularly at the higher temperatures.

Theory also suggests that the rate of heat transmission is dependent to some extent upon the viscosity of the fluid, more particularly if the tube is smooth (see eqn. 11, p. 119). The results in Fig. 101, p. 204 of *Heat Transmission in Boilers, etc.*, for instance, referring to oil-flow, would indicate that the rate is only about one-half of what might be expected from the value of the specific heat. Probably the low conductivity together with the effect of viscosity of the oil is the cause of this reduction in the rate of heat transmission.

Altogether, then, in whatever direction the Reynolds law is tested against the results of reliable experiments, it does not fail to accord in a reasonable manner. Therefore the author has not thought it worth while to attempt to express by formulæ the results of the rates of heat transmission exhibited by the various graphs. The principal graphs of the rates of heat transmission are all collected in Fig. 23, p. 47 of *Heat Transmission in Boilers, Condensers, and Evaporators*, together with the corresponding rates from boiler tests.

The Influence of Deposits on the Temperature of the Metal and on the Transmission of Heat to Boiling Water.—In many of the experiments so far discussed the surfaces of the metal were in a clean condition. In practice, of course, it commonly occurs that the surfaces accumulate a deposit of scale, dirt, or oil. Such deposits generally increase* the total resistance to heat transmission and also have an influence on the temperature of the metal. It was demonstrated on p. 109 that, whereas a deposit of scale, dirt, or oil might not greatly affect the rate of heat transmission in boilers or their efficiency, a similar deposit on the tube surfaces of condensers would have a comparatively large influence on the rate of heat transmission and therefore on the vacuum obtainable. Practical experience on boilers and condensers generally bears out these conclusions.

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* The writer is aware that some have claimed that a deposit may reduce the total resistance. For example, it has been asserted that a thin deposit of varnish on the jacket walls of an internal combustion engine increases the cooling action of the jacket. In effect this would mean that either the resistance to heat transmission of such a smooth deposit is less than that of the supposed films of fluid more or less filling the almost microscopic hollows in the metal, or else that the smooth surface reduces the resistance to the flow of the cooling fluid to such an extent that the reduced resistance to heat-transmission due to the increased velocity more than compensates for the added resistance of the deposit. Also it has been claimed that deposits on the gas side of a metal surface may reduce the total resistance. In such case,

Some interesting experiments were made by Hirsch* on a circular piece of boiler plate, ⅜ in. thick, 15·7 in. diameter, placed over a crucible lined with fireclay, with a pipe at the bottom for a supply of gas and air. The upper edge of the crucible was about ¾ in. distant from the plate, and was encircled by an outer cylinder or casing of fireclay, which served to concentrate the heat and to limit the portion of the boiler plate subjected to it. The gas burned inside the crucible and passed between it and the outer fireclay casing after playing on the bottom of the plate. A copper cylinder was attached to the plate so as to form an open boiler, in which a constant water level was maintained, the water being supplied from a calibrated glass bottle so that the amount of water supplied to the boiler could be measured. This, of course, was a measure of the amount of water evaporated in the same time.

To make an estimate of the temperature of the metal Hirsch inserted plugs of fusible metal of definite melting points into circular recesses drilled in the boiler plate, ⅛ in. diameter and ¼ in. deep. Twelve of these plugs of fusible metal, carefully numbered, were arranged according to the temperatures at which they melted, in a circle 2⅜ in. diameter round the centre of the bottom of the plate. A second series of twelve plugs was disposed round the same centre in a circle 4¾ in. diameter, the diameter of the boiler plate subjected to the flame being

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the actions just mentioned, coupled with the influence of the increased mean temperatures of the gases near the surface on the rate of heat transmission, would more than compensate for the resistance of the deposit to the flow of heat. But until further data is available of an authentic character no useful purpose would be served by further discussion of such conditions.

* *Bulletin de la Société d'Encouragement pour l'Industrie National,* Paris, 1890; or "Experiments on the Overheating of Steam Boilers," *Proc. Inst. Civil Engs.,* Vol. CVIII, 1891–92, Pt. II.

$6\frac{3}{8}$ in. The temperatures at which the fusible alloys melted ranged from 230° F., to lead at 635° F.,* and zinc at 842° F.* It must be remembered, however, that this method of estimating plate temperatures is now considered to be very crude.

The first experiments were made with distilled water and clean plate surfaces at different rates of evaporation. It was found, for example, that when evaporating 29 lb. of water per hour per square foot of heating surface at 212° F. the plate temperature, estimated from the fused plugs, was between 338° F. and 369° F., say, 350° F. This seems to be a much higher temperature than has been obtained since by other experimenters using similar apparatus.

The effect of incrustations on the water side of the plate in boilers was partially imitated by covering the interior surface with a layer of plaster $\frac{1}{25}$ in. thick. The difference of temperature between the plate and the boiling water was found to be about 54° F. higher than when distilled water was in direct contact with the metal. The layer of plaster was then increased to a thickness of $\frac{3}{16}$ in. When evaporating at 212° F. 30 lb. of water per hour per square foot under these conditions, the temperature of the plate was estimated by means of the melted alloys at about 482° F., and when evaporating 40 lb. it was estimated to exceed 752° F.

Other experiments were made to illustrate the effect of flaws in the metal or a badly made joint. Finely powdered talc was strewed over the interior surface of the plate, and a sheet of steel $\frac{3}{16}$ in. thick was then bolted on to the plate till the distance between them was only $\frac{1}{250}$ of an inch.

Distilled water was used, and when evaporating 30 lb. per square foot per hour at 212° F. the temperature of the plate was estimated to exceed 662° F. With an evaporation of 50 lb., all the plugs were melted and the temperature was therefore estimated to exceed 842° F.

Some further experiments were made to examine the effect of a layer of greasy matter on the water side of the plate. The surface was first covered with mineral oil, which was then wiped off, leaving a greasy layer of no appreciable thickness.

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* These are the melting points given by Hirsch. More correct values will be found in Table III, p. 158, of *The Measurement of Steady and Fluctuating Temperatures*.

It was estimated that when evaporating 30 lb. per hour per square foot the flame surface of the plate was 90° F. higher in temperature than when the plate was clean, and when evaporating 50 lb. it was 144° F. higher. In some other experiments under similar conditions with an evaporation of 34½ lb. per hour per square foot all the alloys were melted, the estimation therefore being that the plate exceeded 842° F. in temperature.

In the original paper curves of water evaporation were plotted on the base of plate temperatures under these various conditions. Because of the unreliability of temperature estimations of a plate by means of inserted plugs of alloy it has not been thought worth while to reproduce these curves here.

Hirsch also made some interesting experiments with a saucepan placed over the crucible with the bottom covered with various substances. In one case, for instance, he greased the pan with mineral lubricating oil and heated it without water to decompose it until the bottom was covered with a black grease. After boiling water for a minute, one part of the bottom was found to become red-hot which soon spread over the rest of the surface exposed to the furnace. From the various experiments of this kind Hirsch concluded that—If a perfectly clean tinned iron saucepan is used, however intense the fire the water boils in the usual way and the tin is not melted. Nor is the surface apparently affected by the heat when the saucepan is oxydised or covered with a thin layer of cold mineral oil. But if the oil is previously decomposed by heat, or if a rag smeared with this oil is laid at the bottom of the saucepan, overheating immediately takes place. The same effect is produced if a solution of salt is burned in an oxydised saucepan and smeared with cold mineral oil, even when evaporating only 20 to 24 lb. of water per square foot per hour. Spirits of turpentine or oil of turpentine do not cause overheating unless mixed with a minute portion of linseed oil. It is not so quickly produced with colza as with linseed oil, but is easily produced with red lead. Valvoline when laid on cold only causes a glow when evaporating 70 lb. of water per square foot per hour. Pitch at the bottom of a tinned saucepan floats off when the water boils ; it adheres to an oxidised surface, but does not unduly raise the temperature of the metal. Vegetable grease, such as colza or linseed oil, being easily decomposed by heat, seems to be more dangerous in causing overheating than mineral oil.

An elaborate series of experiments were made by Sir A. J. Durston* when Engineer-in-Chief of the British Navy. The principal object in view was the estimation of the temperatures attained by boiler tube plates, and the means to be adopted to prevent leaky boiler tubes, leakage being ascribed to the effect of excessive tube plate temperatures. The experiments had reference mainly to marine boilers of the return tube type. The first tests were made to ascertain the temperature of the hot side of a plate through which heat was passing to boiling water. For this purpose a circular flanged dish, presumably of steel, 10 in. diameter, 3 in. deep, and ¼ in. thick, had attached to the bottom eight pieces of fusible solder of different compositions, the melting points ranging from 220° F. to 250° F. The vessel was half filled with water and placed over a Bunsen flame, and was allowed to remain until the water had been boiling freely for some time. From the melted alloys it was estimated that the plate temperature was about 240° F., the water boiling at 212° F. Although the rate of evaporation was not stated a comparison with Hirsch's values given on p. 204 evidently shows that Hirsch's plate temperatures under similar conditions were estimated to be much higher. This illustrates the unreliability of the fusible alloy method of estimating plate temperatures.

Further experiments were made with a layer of grease, obtained from the interior of the boilers of a new ship, spread about $\frac{1}{32}$ in. thick over the inside of the bottom of the vessel. The temperature of the hot side of the plate under conditions similar to those in the previous experiments were estimated by the fusible alloys to be about 330° F., or an increase of about 90° F. due to the presence of the grease, with, presumably, about the same rate of evaporation as in the previous case.

Similar experiments were conducted on a larger vessel than before, being a flanged dish, presumably of steel, 2 ft. in diam., 2½ in. deep, and ¼ in. thick. This was placed over a forge fire.

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* "Some Experiments on the Transmission of Heat through Boiler Plates," *Trans. Inst. Naval Arch.*, 1893.

With a moderate blast the temperature of the hot side of the plate when boiling fresh water was about 240° F., as for the previous experiment with a similar dish over a Bunsen flame. More blast was then applied and the temperature went up to about 280° F. This experiment was repeated with various foreign substances in the water, with the following results :—

	Estimated Temperature of hot side of plate °F.
Clean fresh water	280
Mineral oil gradually added up to 5%	310
Fresh water, with 2½% of paraffin	330
Fresh water, with 2½% of methylated spirits	300
*A greasy deposit $\frac{1}{16}$th. in. thick on the plate . . .	above 500

Similar experiments were then made, using a closed vessel, at boiling pressures above atmospheric. With fusible plug alloys as before in the bottom of the vessel the following results were obtained :—

(a) Using clean water and surfaces.

	Estimated temperature of hot side of plate °F.	Temperature of boiling water °F.	Difference °F.
Over Bunsen burner . . .	430	363	67
Over blast forge (full blast) .	430	344·5	85·5

(b) Bottom of vessel coated with grease.

	Estimated temperature of hot side of plate °F.	Temperature of water °F.	Difference °F.
Over forge fire—grease deposit $\frac{1}{16}$ in. thick . . .	510	359	151
,, ,, ,, but using grease of drier or earthier nature . . .	550	351	199
,, ,, ,, and spreading the grease 1½ in. up the sides of the vessel as well, being 1 in. below water level . .	617†	80†	537

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* Other experiments showed that this temperature varied greatly and depended chiefly on the nature and thickness of the deposit.

† These temperatures were reached three minutes after placing the vessel on the fire with full blast turned on.

Some preliminary experiments were then made to determine the temperature of a tube plate at which injurious overheating (i.e. such as to cause leaky tubes) takes place. For this purpose a small boiler was made having 24 tubes, 2½ in. diameter, the tube plate being ⅜ in. thick. Of these tubes 8 were brass, 7 steel, and 9 iron, all expanded in the tube plate. The tube plate was purposely overheated when subjected to an internal steam pressure (without water). By using fusible plugs of lead and zinc screwed into the tube plate, and testing the boiler with a hydraulic pressure of 200 lb. per sq. in. in several experiments after overheating the tube plate, it was estimated that to make the tube joints leak to an appreciable extent the tube plate must be raised at least to the temperature of melting zinc, viz. 750° F.*

Following on this work an experimental boiler was made of the multitubular type, having a built-up brick furnace. There were 65 tubes 2½ in. diameter and length 5 ft. between the tube plates. Of these 27 were iron tubes, 24 plain steel, and 14 steel stay tubes. The tube plates were ¾ in. thick and bolted to the shell at the two ends to facilitate easy removal. Five of the tube holes had each four pieces of fusible alloy ⅝ in. long let into the plate at the middle of its thickness, as shown in the sketch Fig. 96. As first fitted the draught was supplied through a closed ashpit, and a two hours' trial was made under the following conditions :—

	Results.	
	Mean.	Maximum.
Steam pressure, lbs. sq. in.	143	150
Air pressure in closed ashpit, ins. water . .	·3	·5
†Temperature in combustion chamber, °F. .	2850	3100
Temperature in smoke-box, °F.	1400	1600
Coal used per hour, lbs.	188	—
,, ,, ,, per sq. ft. of grate . .	30	—
Water evaporated per hour, lbs. . . .	1039	—
,, ,, ,, per sq. ft. of tube and tube plate heating surface . . .	4·62	—

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* By a Le Chatelier pyrometer (probably platinum, platinum-rhodium). Refer to p. 18, respecting the difficulties of measuring high temperatures of gas.

† See note p. 204.

According to the fusible plugs it would appear that the temperature at the middle of the tube plate was, on the average, about 500° F. in the above test.

The boiler was now enclosed in an air-tight stokehold and the draught increased by using a larger fan. In addition to the fusible alloys inserted into the middle of the plate at the five chosen tubes, four pieces $\frac{5}{32}$ in. in length and $\frac{3}{16}$ in. diameter were fitted into the face of the tube plate around each of the tubes mentioned. These pieces projected $\frac{1}{32}$ in. from the plate. The following table shows the results obtained in four tests with this arrangement:—

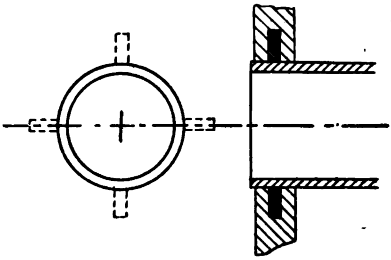

FIG. 96.—Fusible plugs in tube plate to measure temperature, Durston's experiments.

	No. of Test.			
	1	2	3	4
Duration of test, hours	5	5	5	$3\frac{5}{6}$
Pressure of steam, lbs. sq. in.	145	142	140	144
Air pressure in stokehold, ins. water	3	3 for 2 hours 3½ for next 3 hours.	3	2·9
Coal per sq. ft. of grate per hour, lbs.	90	102	84·2	—
Water evaporated per sq. ft. of tube and tube plate heating surface per hour, lbs.	12·64	13·22	11·76	11·99
Temperature in combustion chamber °F	2750	2500	3100	3200
Amount of mineral oil used, lbs.	—	—	9	5*
Oil used in percentage of feed	—	—	·07	·05

On the first test sixteen of the fusible plugs in the face of the plate were made with melting points from 490° F. to 690° F. and the remaining four were of antimony (melting point 1060° F.). All were melted except the antimony.

On the second test the plugs round each tube were one of antimony (1060° F.), two of zinc (750° F.), and one alloy (690° F.). Of these the five antimony and three of the zinc plugs remained intact; all the rest melted.

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* Boiler not cleaned from previous test.

In the third test the five antimony and one out of the ten zinc plugs remained intact, all the rest melted.

On the fourth test, which was a continuation of the third, an additional 5 lb. of oil was admitted with the feed water, but when burning down for the purpose of cleaning the fires after running $3\frac{5}{6}$ hours, the tubes gave out. Around three of the tubes all the zinc and the alloy plugs in the face of the plate melted. The antimony plugs partly melted in two cases, but remained intact in the other three. The zinc alloys remained intact in two.

On withdrawing the tubes the plugs let into the middle of the plate were examined, these ranging from 480° F. melting point to zinc at 750° F. All were found to be melted except the zinc at two tubes.

After the last test, on applying water pressure it was found that at 60 lb. practically all the tubes in the boiler leaked to a greater or lesser extent.

It is sometimes stated that leakage of tubes is due to the chilling action of cold air entering the furnace when the fire door is open. At the conclusion of the second test the fan was kept going for some time after drawing the fire but no leakage of tubes occurred.

Considerable trouble had been experienced up to this time by the leakage of tubes under service conditions in the Navy, particularly with forced draught. Ultimately ferrules were inserted into the hot ends of the tubes, leaving a small air space between the ferrule and the tube and plate. These were effective in reducing the tube end and plate temperatures, but were liable to get burned away.

In the discussion of Sir A. J. Durston's paper Mr. James Howden asserted that on his system of forced draught, in which cold air is prevented from entering the furnaces when the fire doors are opened, not a single case had occurred of leaky tubes in marine boilers, even under comparatively large forced draughts. He therefore concluded that the cause of leaky tubes was the sudden chilling action on the combustion chamber due to cold air entering the furnaces when the fire doors were open.

In any case, and although the fusible plug method of estimating temperatures is subject to grave suspicion, it is fairly evident from the above experiments that the presence of oil in the feed water contributed to the serious leakage of the tubes.

Some interesting experiments have been made by E. M. Bryant* on the rate of heat transmission through steel and copper plates to water boiling at atmospheric pressure. The apparatus used is illustrated in Fig. 97. The inner vessel A had for its base the plate on which the experiments were to be made. On the "guard-ring" principle this vessel A was placed into the central aperture of the vessel B, a lining of non-conducting material being placed in the place between the two vessels in order to prevent flow of heat from the plate to the water in the outer vessel B. The division between the two vessels was extended by means of the mica cylinder *b*, which was double, so that any water projected against it would be returned to the vessel from which it came. The lid or cover formed a steam jacket and the steam passed to a condenser through the opening at *a*. The syphon *c* connected the inner vessel A with a beaker of water, and the level was kept constant by means of a Mariotte bottle so that the water evaporated from the vessel A was given by the fall in height of the water in the bottle. A mercurial thermometer *d* was placed in the syphon to measure the temperature of water entering the inner vessel.

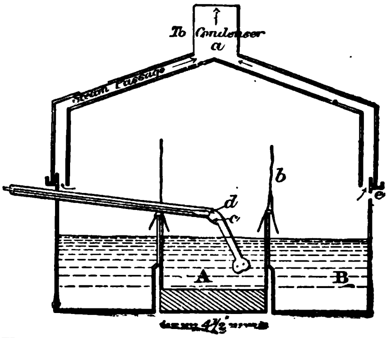

FIG. 97.—Experiments on temperature of boiler plates by E. M. Bryant.

The furnace for heating the plate was formed of a hollow iron hemisphere on which the boiler rested with an asbestos washer between. This was surrounded by a firebrick cylinder and two Fletcher burners were arranged to project the flames on to the outside of the iron hemisphere tangentially so as to heat it uniformly. The hot-iron hemisphere then radiated heat to the plate under test. The temperature of the plate was measured by means of thermo-junctions embedded in the metal. The particular arrangement finally adopted so as to cause the least disturbance to the flow of heat through the metal is described on p. 125 of *The Measurement of Steady and Fluctuating Temperatures*.

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* "On the Thermal Condition of Iron, Steel, and Copper when acting as Boiler Plate," *Proc. Inst. Civil Engs.*, Vol. CXXXII, 1897–98, Part II.

From the temperature measurements at the various depths in the steel or copper plates the temperatures of the two surfaces were deduced and were tabulated in the paper. In Fig. 98 the writer has plotted these temperatures for the

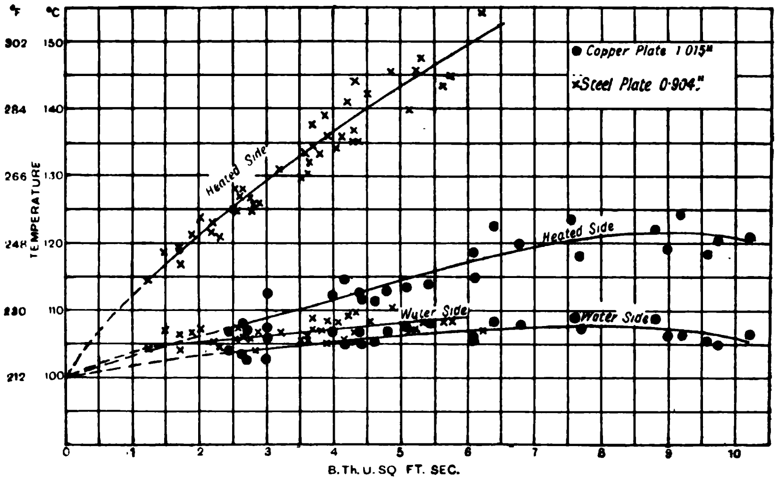

Fig. 98.—Temperatures of steel and copper plates in Bryant's experiments.

steel and copper plates on the base of heat transmission per square foot of surface of plate per second. According to the values shown it is seen that for the clean copper plate the temperature on the water side increased gradually up to a maximum value in the neighbourhood of 8 B.Th.U. per sq. ft. per sec. corresponding to an evaporation of about 30 lb. of water per sq. ft. per hour from and at 212° F. The probable reason for this maximum is the increase in the rate of circulation with increasing evaporation. The difference between the temperatures on the fire and water side increases practically in proportion to the heat-flow.

For the clean steel plate the temperature on the water side appears to increase slowly with the heat-flow within the range of the experiments. The greater difference of temperature between the two sides of the steel plate, as compared with the copper plate, is well illustrated.

A curious effect was noticed during the experiments. On several occasions when the vessel boiled dry, a sudden fall of the temperature of the plate occurred, especially near its upper surface, followed by a rapid rise. The cooling was probably due to the rapid movement or circulation taking place when the water was nearly boiled away, followed by a rise of temperature as soon as the surface became dry. This result would appear to confirm the fall of the temperature on the water side of the copper plate shown in Fig. 98 beyond the value of the heat-flow of about 8 B.Th.U. per sq. ft. per sec.

A number of experiments were made on the effect of a film or coating of oil on the water side of the plate. The plate was dried, well oiled with sperm oil, vaseline, best sweet oil, and gas-engine oil, and the water then put in the vessel. Most of the oil came to the surface, and if a film remained on the surface of the plate it was not sufficient to produce any appreciable effect in raising the temperature. After this the plate was oiled and then heated to about 200° C. (392° F.) before the water was put in. The oil turned brown but had no more effect than before. A thin layer of lime deposited on the plate had no appreciable effect on the temperature of the surface, but when oiled with gas-engine oil the surface temperature rose about 2° C. higher than it would have been had it been clean. When the surface was oiled with gas-engine oil or sperm oil, and then heated up to 200° C. the temperature of the plate rose some 4° C. Such variations, however, were not much greater than occurred when the surface was kept clean.

Linseed oil, sperm oil, and best sweet oil were each then treated in the following manner. A small quantity of the oil was put in the boiler and heated till it decomposed and a dark brown waxy layer was formed on the surface of the plate having a thickness of about $\frac{1}{100}$ in. The maximum rise of temperature which occurred when boiling water would be about equivalent to the resistance of ·7 in. thickness of iron. When transmitting 4 B.Th.U. per sq. ft. per sec. this thickness is equivalent to a rise of about 20° F. in the temperature of the metal.

When heating the water up to boiling-point the indications of a thermo-junction in the plate near the lower surface were noted in some of the experiments. The temperature of the metal plate was found to rise steadily until boiling occurred and reached a steady state when boiling had continued for some time. This result is quite contrary to some experiments made by other observers. For example, Professor A. H. Gibson* made some experiments on an open cast-iron vessel 8 in. diameter and heated by gas-jets. Keeping the water well stirred and using a platinum-iridium thermo-junction he estimated that the metal in the water side rose to 220° F. when the water was at 180° F. and then fell to nearly the boiling temperature with the water boiling.

Some further experiments were made by Bryant to compare plugs of fusible alloy inserted in the plate with the thermo-junctions. Hirsch's method of inserting the plugs, and of finding their melting points, was carefully followed, but the alloys used were not found to melt at definite temperatures. For this reason measurements were only possible within somewhat wide limits. The following results show, however, that the temperatures estimated by the fusible plugs were in excess of those given by the thermo-junctions :—

Temperature of Fire-side of Plate	
By thermo-junction, °C.	By fusible plugs, °C.
112·5	About 123°
123·5	Between 130° and 149°
130·0	Above 161°

Instructive experiments have been made by L. Austin† at the Physikalisch-Technischen Reichsanstalt. The apparatus used is shown diagrammatically in Fig. 99. A flat iron wall I, 3 cm. (1·18 in.) thick was fixed between the two vessels shown, one vessel containing oil and the other water. The length of the vessels was 26 cms. (10·2 in.) and the width sufficient to accommodate the stirrers as shown, these having four blades each. Both stirrers could be rotated by the same electric motor for the purpose of circulating the fluids. The water chamber was nearly filled with water and closed tightly by the cover D with the rubber packing G. The vapour formed when the water was boiling was led to two condensers R. The water coils K served to cool the water in the vessel when required. The oil vessel was first charged with lubricating oil, but later rape-seed oil was used. The oil could be heated up to 300° C. (572° F.) by Bunsen burners placed well away from the plate I.

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* *Trans. Engs. and Shipbds. in Scotland*, 1910–11.

† "Ueber den Wärmedurchgang durch Heizflächen," *Zeit. d. Verein Deutscher Ingenieure*, 13th Dec., 1902, Vol. XLVI, p. 1890; or *Engineering*, 1st Jan., 1904, Vol. LXXVII, p. 1.

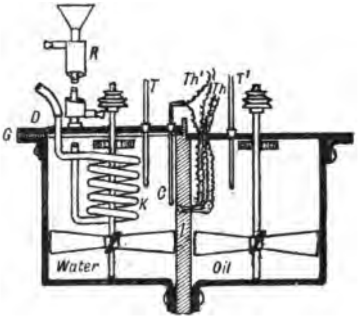

Fig. 99.—Apparatus used in Austin's experiments on heat transmission.

Two thermo-couples were used, presumably of copper-constantan, to measure the wall temperatures. The couple Th. measured the gradient of temperature through the wall. Two holes, 1 mm. diameter, were bored to accommodate this thermo-couple Th., one at a depth of 2·555 cm. (1·002 in.) and the other at a depth 1·51 cm. (·595 in.). The junctions in these two holes, being in the same circuit and in opposition, enabled Th. to give the difference of temperature between these two points in the metal, and hence the temperature gradient was easily calculable. The other couple Th. measured the difference of temperature between a point in the metal 2·6 cm. deep and the water at C, as is indicated in Fig. 99.

The temperature gradient q in the metal gave a measure of the heat-flow, and also the temperature of the metal at the two surfaces could be calculated. The conductivity k had been determined for the same iron by Jaeger and Diesselhorst, who gave the value ·148 gr.-cal. per sq. cm. per sec. per °C.

difference of temperature in a thickness of 1 cm., equivalent to nearly ·01B.Th.U. per sq. ft. per sec. per °F. per ft. thick.

Experiments were first made with the heat flowing through the iron plate from the oil to the boiling water (100° C.) without stirring the water and then running the stirrer at about 200 revs. per min. The temperature difference, $\triangle$, between the wall and the water is shown plotted in Fig. 100 on the base of B.Th.U. transmitted per square foot per second. It will be noted that the stirring of the boiling water caused an appreciable reduction of the temperature difference. Later experiments were made with the flow of heat from the boiling water to the oil through the metal, both with and without stirring. The results are also shown plotted in Fig. 100, and it is seen that, within the limits of the experiments, there was little or no difference between the values whether the heat was flowing from the iron plate to the water or *vice versa.*

The ratio $\frac{\triangle}{q}$ gives a measure of the surface resistance to heat-flow between the metal and the water in an equivalent thickness of iron. These have also been plotted in Fig. 100 in inches of iron. It will be seen that, whereas when stirring the boiling water the surface resistance was about ·3 in. of iron, when not being stirred the resistance fell gradually from about 1 in. to about ·4 in. of iron.

These results indicate that the rate of circulation of boiling water has an influence on the surface resistance, since with rapid stirring the resistance is practically constant, but without stirring the resistance fell as the rate of evaporation, or the rate of heat-flow, increased and consequently as the rate of circulation increased. Unfortunately, however, these results are only comparative because there were no means of indicating the actual velocity of the water relatively to the heating surface.

Some further experiments were made with the water at various temperatures (not boiling) and with the water stirrer running at 200 revs. per min. The results are shown plotted in Fig. 101. It is seen that the surface resistance decreases as the temperature of the water increases, and apparently decreases slightly with the increase of heat-flow. The line OA for boiling water, with the stirrer running at about the

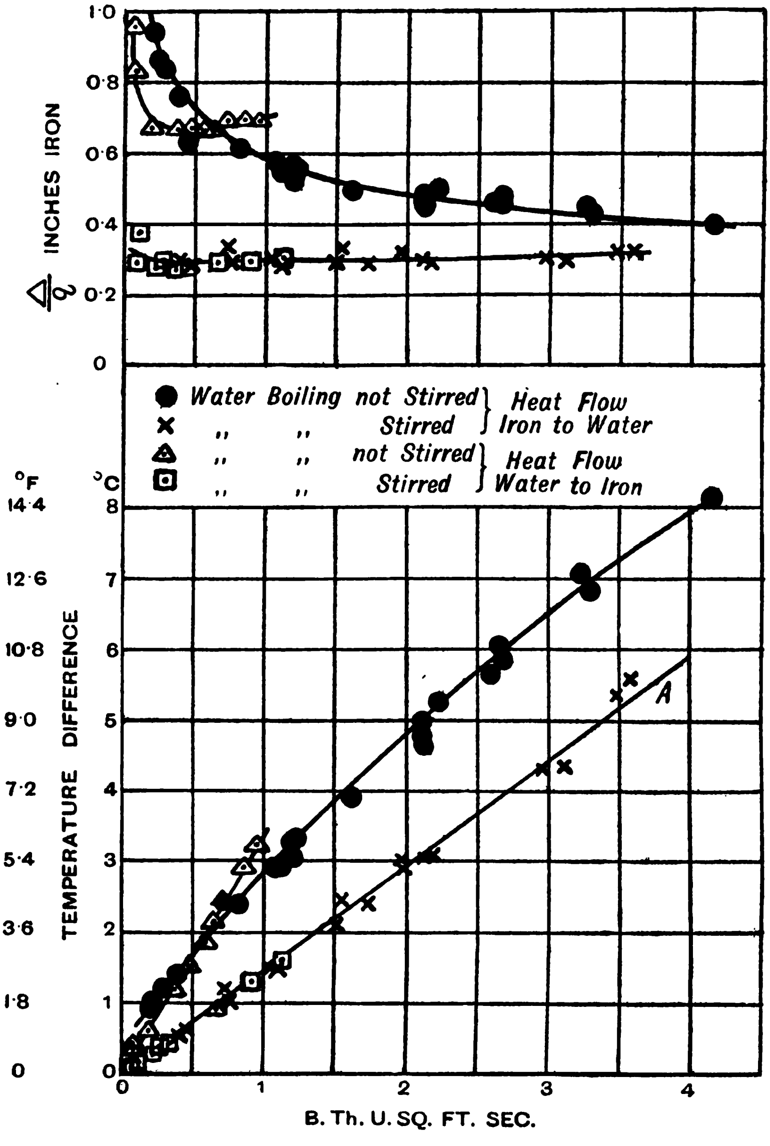

FIG. 100.—Difference of temperature between metal and boiling water obtained from apparatus in Fig. 99.

same speed, is copied from Fig. 100. It would appear to indicate that, with a fairly rapid rate of circulation, there is little

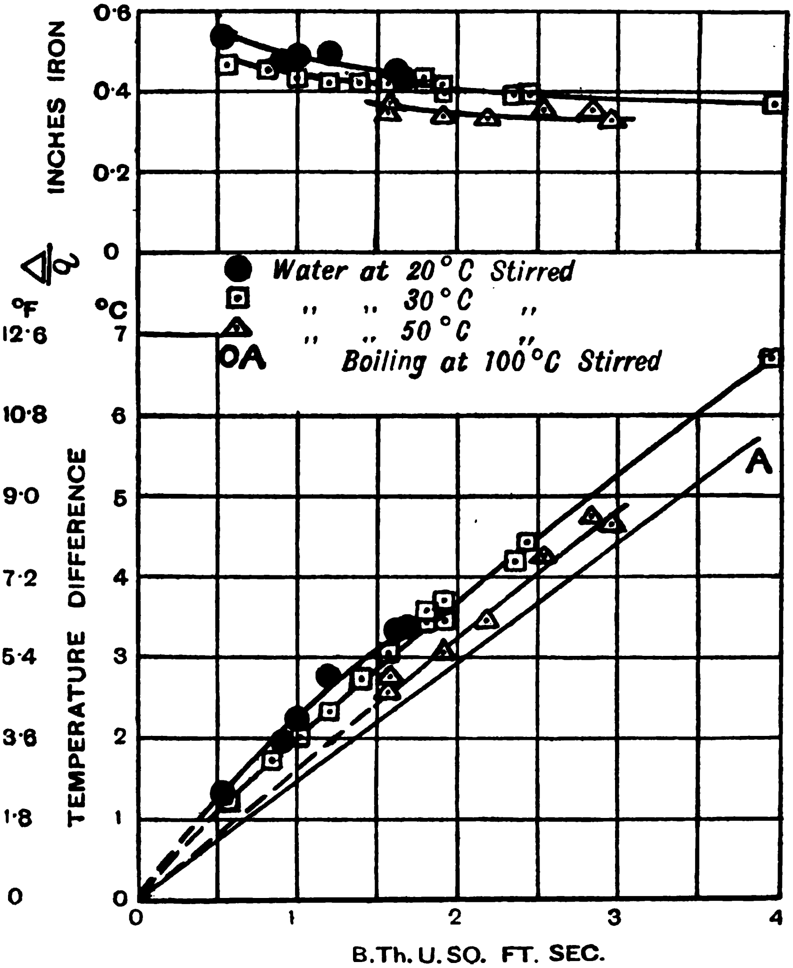

FIG. 101.—Difference of temperature between metal and water obtained from apparatus in Fig. 99.

or no difference between the surface resistance for boiling and non-boiling water at a given temperature.

This is also indicated from the results of experiments made with water at 30° C., 90° C., and boiling at 100° C., with different speeds of the stirrer. As shown in Fig. 102, above about 100 revs. per min. there was practically no difference between the resistance of non-boiling water at 90° C. and water boiling at 100° C. It will also be noted that with water being heated at 30° C. without stirring, the surface resistance was as much as 4 in. of iron, whereas, with boiling water at 100° C. the rate of circulation was such that even without stirring the resistance was only about ·5 in. of iron.

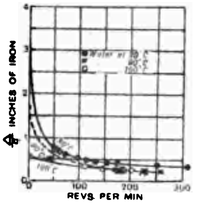

FIG. 102.—Surface resistances between metal and water at different speeds of stirrer in Fig. 99.

Surface conditions seemed to make no appreciable difference to the resistance, for after carefully cleaning the surface the results were practically the same. Apparently an attempt was also made by the experimenter to use revolving brushes to sweep the surface during an experiment but without success.

With respect to the flow of heat from the oil to the iron surface it was stated that with rape-seed oil at 90° C. and the iron surface at 50° C. the surface resistance on the oil side with the stirrer running at about 100 revs. per min. was found to be equivalent to 18 cm. (7·09 in.) of iron, and with this oil at 156° C. and a surface temperature 120° C. it was equivalent to 8 cm. (3·15 in.) of iron. With lubricating oil the values were higher, 30 cm. (11·8 in.) and 13 cm. (5·12 in.) being observed.

Preliminary to these experiments by Austin similar experiments were made by Holborn and Dittenberger* at the Reichsanstalt with an open circular cylinder of brass 21 cm. high, and 4·05 cm. and 6·05 cm. inner and outer radius respectively. This cylinder had a hemispherical bottom and it was placed in a heated oil bath. Thermo-couples similar to those previously described were used to measure the gradient of temperature in the metal and the temperature difference between the metal and the water inside the vessel. Although the results, where comparable, were not quite coincident with those of Austin's, they were of the same order of magnitude, and it is hardly worth while to discuss them in detail.

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* "Ueber den Wärmedurchgang durch Heizflächen," *Zeit d. Ver. Deut. Ing.*, Vol. XLIV, p. 1724.

Comparing Austin's results, Fig. 100, with those of Bryant, Fig. 98, as regards the values of $\triangle$, the temperature difference between boiling water and the plate, it is seen that the results are much the same where comparable.

Some experiments made on the temperature of the tubes in a water-tube boiler are recorded in the Technical Paper 114, issued by the Bureau of Mines, Washington. These experiments, by Kreisinger and Barkley, were made on the Heine water-tube boiler represented in Fig. 19, p. 42, of *Heat Transmission in Boilers, etc.*, except that the furnace and ashpit door were sealed and the top of the bridge wall was tight against the lower row of tubes. The hot products of combustion from an experimental furnace, having a combustion chamber 40 ft. long, were delivered to the boiler at the back end, entering among the tubes at the opening there. Baffles were also introduced so that the gases made two passes along the length of the tubes and finally left the tubes at the back end. Several copper-constantan thermo-couples were embedded in the metal of a tube in the bottom row at a point where the hot products of combustion entered among the tubes. One couple was also placed in a tube in the highest row where the gases left the boiler. The detailed construction of these thermo-couples is described on p. 126 of *The Measurement of Steady and Fluctuating Temperatures.* Table 23 gives values illustrating the character of the results obtained from experiments with the tubes practically clean, and an inspection and comparison of the values

TABLE 23

Measured inlet gas temperature °F.	Tube temperature at gas inlet.			Column 3 minus temperature of boiler water °F.	Estimated B.Th.U. per sq. ft. per sec. at gas inlet.	Estimated B.Th.U. per sq ft. per sec. per deg. Fah. between tube and water at gas inlet.	Estimated average B.Th.U. per sq. ft. per sec. for whole boiler.
	Gas side °F.	Water side °F.	Difference °F.				
1	2	3	4	5	6	7	8
2100	399	360	39	12·5	27·3	2·18	·46
2460	415	361	54	13·5	37·8	2·80	·86
2550	404	358	46	14·5	32·2	2·21	1·18

in columns 5 and 6 with Bryant's results in Fig. 98, p. 212, and Austin's results in Fig. 100, p. 217, show that the difference of temperature between the tube and the water was similar to those in Bryant's and in Austin's experiments.

Column 6 of Table 23 was derived from the measured difference of temperature between the two sides of the tube ·12 in. thick, using ·007 as the conductivity of the metal, in lb., ft., sec., deg. Fahr. units. Column 8 was obtained from the measured rate of evaporation for the whole boiler.

Very little reliable data is available respecting the actual conductivity of scale deposits on boiler plates or on condenser tubes. It is known, however, that the conductivity of scale varies greatly with the porosity and density of the scale. Experiments have been made by W. E. Ernst.* Plates of scale were held between copper plates and the temperatures of the plates were measured. In order to obtain good surface contact glycerine was squeezed between the plates, an allowance for the glycerine being made in the calculations. Ten different scales were tried and their conductivities ranged from ·00021 to ·00052 B.Th.U. per sq. ft. per sec. per ft. thick per deg. Fahr., depending upon the porosity and density of the scale. The conductivity of fats, oils, coal-tar, and anti-scale compounds were also determined, giving values ranging from ·0000195 to ·0000218 to the same units, which would indicate that the resistance of a given thickness of such deposits is more than ten times that of a deposit of scale of the same thickness.

These experiments, however, do not really imitate the conditions when a boiler plate is covered with scale in contact with water, for in the latter case with a porous scale the water percolates through the scale up to the plate and it is impossible to state where the water really becomes heated or evaporated. Thus the conductivity of a scale is not always a measure of its effect on the heat transmission, as it may depend largely upon the wetness and porosity of the scale in a boiler.

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* "Conductivity of Scale and other Incrustations on Boiler Heating Surfaces," summarised by Professor W. E. Dalby in his paper on "Heat Transmission," *Proc. Inst. Mech. Engs.*, Oct.–Dec., 1909, p. 940.

Experiments have been made by Eberle* on the temperature of a plate with deposits of scale or of asphalt when boiling water at 100° C. The plate formed the bottom of a tank filled with water and the scale was formed by boiling the water for many days and weeks, first with sulphate of lime and other ingredients to form a scale 1·48 mm. (·057 in.) thick and then with magnesium carbonate to give a total thickness of about 5 mm. (·197 in.). The plate was heated indirectly by allowing a gas burner to heat a plate a little below the boiler plate under test.

When transmitting about 3·1 B.Th.U. per sq. ft. of boiler plate per sec. with boiling water at 100° C. (212° F.) the following results were obtained :—

Conditions.	Temperature of Plate.		Excess over boiling water.	
Clean plate, water boiling at 100° C.	130° C.	266° F.	30° C.	54° F.
Scale ·2 in. thick do. do.	188	370	88	158
Thin asphalt do. do.	270	518	170	306
Asphalt, ·012 in. thick do. do.	510	950	410	738

With the scale ·2 in. thick, taking the temperature of the scale in contact with the water to be 266° F., the conductivity comes out about ·0005, which compares fairly well with the values deduced by Ernst on a different apparatus.

Corollaries to the Reynolds Law of Heat Transmission. — The Reynolds law of heat transmission was deduced by the late Professor Osborne Reynolds from a consideration of the molecular motions involved in the flow of fluids through tubes. The writer would suggest that similar laws are applicable to the conditions mentioned below and discussed more fully later. These suggestions, however, are more or less speculative in character until subjected to experimental proof. These corollaries refer to the following subjects :—

1. The rate of heat transmission from the gases to the walls of a gun.

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* *Zeit. d. Ver. Deut. Ing.*, 1910; or *Engineering*, Vol. LXXXIX, 1910, p. 655.

2. The rate of chemical reaction in fluids flowing through a channel or tube.
3. The rate of cooling of liquids by evaporation.
4. The rate of sterilisation of fluids by the application of heat.

Corollary 1. *The Rate of Heat Transmission from the Gases to the Walls of a Gun.*—If the Reynolds law of heat transmission holds at the pressures and temperatures of the gases discharged from a gun when fired, then the rate of heat transmission h_1 due to convection should be nearly proportional to

$$v_1\rho_1 = \frac{w_1}{a_1}$$

where w_1 is the weight of gases discharged per unit of time and a_1 the area of cross-section of the bore. In other words, this would mean that although a gun is fired intermittently at definite intervals of time, the amount of heat transmitted to the walls per unit of area and time due to convection of the gases will be nearly proportional to $\frac{w_1}{a_1}(\mathrm{T}_g-\theta)$, where $(\mathrm{T}_g-\theta)$ represents the average difference of temperature between the gases and the metal surface of the gun. Of course there would be an additional flow of heat due to radiation from the gases and also to other incidental circumstances such as that caused by the frictional resistance between the shot and the walls at discharge. So far as the writer is aware no definite experiments have been made to ascertain how far the Reynolds law is applicable to such conditions.

Corollary 2. *The Rate of Chemical Reaction in Fluids flowing through a Channel or Tube.*—When reagents having chemical affinity are brought together it is well known that, in general, the more rapidly or completely they are mixed together the more rapidly does chemical combination take place. This is equivalent to saying that the rate of combination depends to some extent upon the rate at which the molecules of the reagents are dispersed or diffused among one another. Thus with fluids, if the reagents were allowed to flow together through a channel or tube at velocities well above the critical value, the convective character of the flow would ensure the rapid dispersion of the molecules. But the resistance to the flow through a tube is due to the exchange of momentum between the molecules as they move about from place to place,* which suggests that some relation exists between the rate of flow, the resistance to the flow, and the rate of chemical combination, similar in character to the Reynolds law of heat transmission between fluids and a surface. No doubt the chemical affinities and physical properties of the reagents are also determining factors as well as the state of subdivision existing when the reagents enter the channel or tube.

So far as the writer is aware no definite experiments are available which could be used to test this theory, but its importance would be recognised when it is mentioned that it would be applicable to such problems as the rate of combustion in a furnace through which gases are flowing, or the rate at which solid substances can be dissolved by fluids flowing past, or the rate at which a fluid may be absorbed or entrained by another suitable fluid, or the rate of absorption of vapours by gases dealt with in corollary 3.

COROLLARY 3. *The Rate of Cooling of Liquids by Evaporation.*—A common illustration of the cooling of a liquid by the evaporation of some of the liquid is afforded by the cooling towers sometimes used for the cooling of condensing or circulating water in large power stations. The hot water, in descending through the tower, is brought into more or less intimate contact with air as it ascends through the tower. The air takes up vapour and the latent heat required for the evaporation is supplied by the remainder of the water in the neighbourhood of the evaporation, and the water therefore cools in consequence. The principles involved are discussed at some length on p. 215 of *Heat Transmission in Boilers, Condensers, and Evaporators.*

For the present purpose it would be noted that the Reynolds law of heat transmission, being based upon molecular convection, would suggest that a similar law should hold with respect to the rate at which a flowing gas can absorb vapour. It is fairly certain, then, that the greater the surface of contact, the more rapid the relative motion of the gas and the liquid,* the greater the difference of temperature, and the smaller the amount of vapour already associated with the gas,* the greater would be the rate of absorption of the vapour at any particular temperature.

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* This is only true for a gas ; with liquids the force of cohesion may play some part in the resistance offered to the flow.

COROLLARY 4. *The Rate of Sterilisation of Fluids by the Application of Heat.*—One method of sterilising or Pasteurising fluids is to kill the bacteria or other micro-organisms by raising them to a sufficiently high temperature. For example, milk can be Pasteurised by raising the temperature of the milk to 80° C. and keeping it at this temperature for a sufficient length of time.

Now the time required to raise the temperature of a fluid to a particular value by the application of heat at a surface would depend upon the rate of heat transmission and on the difference of temperature between the fluid and the surface, more or less according to Reynolds law.

After the fluid reaches the desired temperature some time would be necessary for the individual bacteria to acquire this temperature, because it is probable that capillary attraction would cause each of them to be surrounded by a tenacious film of relatively stationary fluid through which heat can only be transmitted by conduction (excluding from the present discussion the small amount of radiation which might occur). But the more rapidly the organisms are made to move relatively to the body of fluid the greater would be the rate of heat transmission between the body of fluid and the organisms and the more rapidly would they attain the same temperature as the fluid. It is probable, therefore, that the higher the velocity of flow the greater would be the rate at which these organisms would be killed, but to what extent this holds can only be determined by carefully conducted experiments.

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* The influence of the velocity of the gas and the humidity of the air are discussed to some extent on p. 233 of *Heat Transmission in Boilers, etc.*, in connection with cooling towers, cooling ponds, and evaporative surface condensers.

TABLES AND DATA

TABLE 24

THERMAL CONDUCTIVITY

Substance.	Temperature °F.	Conductivity B.Th.U. per sq. ft. per sec. per °F. per ft. thick.	Authority.
Aluminium	32 212	·023 ·0243	Lorenz
Brass	—	·0206	Neumann
,,	—	·0101	Weber
,,	77 to 95	·0181	Lees
,, (yellow)	32 212	·0137 ·017	Lorenz
,, (red)	32 212	·0165 ·019	,,
Copper	—	·0743	Neumann
,,	32 212	·0482 ·0485	Lorenz
,,	—	·0336 ± ·0101	Bryant
,,	t	·0709 + ·0000524 t ·0465 + ·000037 t	Tait
,,	t	·0715 − ·000082 t ·0678 − ·0000556 t	Ångstrom
,,	t	·0765 − ·0000418 t	Stewart
,,	Between 50 and 207	·0644 to ·0579	J. H. Gray
,, (very impure)	Between 50 and 207	·0234 to ·0214	
Cast-iron	100 to 350	·0077 to ·00709	Callendar and Nicolson
Iron	32 212	·0112 ·0109	Lorenz
,,	—	·011	Neumann
,,	—	·00995	Jaeger and Diesselhorst
,,	t	·0144 − ·00001145 t	Forbes
,,	t	·0132 − ·00000015 t	Tait
,,	t	·0121 − ·0000098 t	Stewart
,,	t	·0140 − ·0000213 t	Ångstrom
Lead	32 212	·00561 ·00512	Lorenz

TABLE 24—(*continued*)

THERMAL CONDUCTIVITY

Substance.	Temperature °F.	Conductivity B.Th.U. per sq. ft. per sec. per °F. per ft. thick.	Authority.
Nickel	64	·00953	Jaeger and Diesselhorst
Platinum	64 212	·0112 ·01163	—
Silver	32	·0735	Weber
Steel (hard)	—	·00416	Kohlrausch
,, (mild)	—	·00745	
,, ,,	about 140	·00805	Callendar & Nicolson
,, plate	— —	·0066 ± ·000735 ·0064 ± ·000403	Bryant
Tin	32 212	·0102 ·00955	Lorenz
Zinc	64	·0171	Jaeger and Diesselhorst
,,	—	·0206	Neumann
Asbestos	32 212 572 1112	·0000242 ·0000312 ·0000347 ·0000380	Nusselt
Asbestos paper	—	·0000288	Lees and Chorlton
Asbestos sheet (½ in. thick)	—	·0000466	Bacon
Asbestos millboard (⅜ in. thick)	—	·0000261 to ·0000186	—
Blotting paper	—	·000010	Lees and Chorlton
Cork	32	·0000481	G. Forbes
,, (powdered)	32 212 392	·00000579 ·00000796 ·0000103 (sp.gr. ·162)	Nusselt
,, (asphalted)	64	·0000114 (sp. gr. ·2)	,,
,, (compressed slabs)	122 to 167	·0000112 (sp. gr. ·166)	Bacon
Cotton wool	32	·0000029	G. Forbes
,, ,, (pressed)	32	·0000022	
,, ,,	32 212	·0000088 ·000011 (sp. gr. ·081)	Nusselt
Ebonite	120	·0000248	Herschel, Lebour & Dunn
Fats, oils, coal tar, anti-scale compounds	—	·0000195 to ·0000218	Ernst

TABLE 24—(*continued*)

THERMAL CONDUCTIVITY

Substance.	Temperature °F.	Conductivity B.Th.U. per sq. ft. per sec. per °F. per ft. thick.	Authority.
Felt	32	·0000058	G. Forbes
Firebrick	—	·000118	Neumann
,,	1290	·00016	Clement and Egy
,,	—	·00013	Landolt and Börnstein
Fireclay brick	brnd. at 1920 ,, ,, 2370	·000246 ·000338	Wologdine
Flannel (dry)	32	·0000081	G. Forbes
Infusorial earth (loose)	212 572	·0000123 ·0000145 (sp.gr. ·35)	Nusselt
,, ,, (baked)	212 572	·0000145 ·0000198 (sp. gr. ·20)	,,
,, ,, (block)	302 662	·0000155 ·0000230 (sp. gr. ·58)	,,
Plaster of Paris	—	·000047	Lees and Chorlton
,, ,, ,, (powder)	—	·00017	
Portland cement	—	·000048	,, ,, ,,
Sawdust	—	·0000081	G. Forbes
,,	122	·0000102	Nusselt
Scale (boiler)	—	·00021 to ·00052	Ernst
,, ,,	—	·0005	Eberle
Slate	—	·000242	Lees and Chorlton
Slagwool or silicate cotton	122 to 167	·00013	Bacon
Wood (fir, parallel to axis)	—	·00002	G. Forbes
,, (fir, perpendicular to axis)	—	·000006	
,, (yellow pine across grain)	—	·0000242	Bacon
,, (teak, across grain)	—	·000043	
Wool (sheep), greasy and not cleaned	32 212	·00000615 ·00000930 (sp.gr. ·136)	Nusselt

TABLE 24—(*continued*)

THERMAL CONDUCTIVITY

Substance.	Temperature °F.	Conductivity B.Th.U. per sq. ft. per sec. per °F. per ft. thick.	Authority.
Glycerine	48 to 59	·000046	Weber
Oils, Olive	—	·0000265	Wachsmuth
,, Castor	—	·0000285	
,, Petroleum	55	·0000238	Graetz
,, Vaseline	—	·0000295	Lees
Sodium chloride solution (33½ %)	55	·000179	Winkelmann
Water	32	·0000805	Weber
	53	·0000912	
,,	64	·0000832	Chree
,,	68	·0000963	Chattock and Milnes
,,	77	·0000915	Lees
	116	·0000805	
Air	32	·0000038	Winkelmann
,,	—	·00000374	Stefan
,,	—	·00000322	Kundt and Warburg
Carbon-dioxide	32	·00000206	Winkelmann
Hydrogen	32	·0000219	,,
,,	—	·0000228	Kundt and Warburg
Nitrogen	45	·00000351	Winkelmann
Oxygen	45	·00000376	,,

TABLE 25

SPECIFIC HEAT

Substance.	Temperature °F.	Specific heat.	Authority.
Aluminium	212	·2226	Boutschew
	482	·2382	
	932	·2739	
Brass (red)	32	·0899	Lorenz
,, (yellow)	32	·0883	,,
80 Cu. + 20 Sn.	57 to 208	·0862 (mean)	Regnault
88·7 Cu. + 11·3 Al.	68 to 212	·0946 ,,	Luginen
Copper	63	·0924	Naccari
	212	·0942	
,,	59 to 460	·0951 (mean)	Magnus
,,	1652	·1259	Richards
Gold	73 to 212	·0940 (mean)	Trowbridge
,,	32 to 212	·0316 ,,	Violle
Iron (cast)	100 to 350	·115 to ·130	Callendar and Nicolson
,, (wrought)	59 to 212	·1152 (mean)	Nichol
	1832 to 2192	·1989 ,,	,,
Lead	212	·0311	Naccari
	572	·0338	
,,	64 to 212	·03096 (mean)	Magnus
,,	61 to 493	·03191 ,,	,,
Mercury	32	·03346	Barnes and Cook
	185	·03280	
,,	212	·03284	Naccari
	482	·03212	
Nickel	212 to 1832	·1128 to ·1608	Pionchon
Platinum	212	·0275	Tilden
,,	932 to 2732	·0356 to ·0390	White
Silver	73	·055	Naccari
	212	·0566	
,,	932	·0581	Tilden
Tin (cast)	70 to 228	·0551	Spring
Zinc	212	·0951	Naccari
	392	·0996	
	572	·1040	

Glass (crown)	50 to 122	·161 (mean)	Meyer
,, (flint)	50 to 122	·117 ,,	,,

TABLE 25—(*continued*)

SPECIFIC HEAT

Substance.	Temperature °F.	Specific heat.	Authority.
Paraffin	32 to 68	·694 (mean)	Weber
,,	95 to 104	·622 ,,	Batelli
,, (fluid)	140	·712	,,
Sea water, sp. gr. 1·0043	63·5	·98	Thomsen
Sea water, sp. gr. 1·0235	63·5	·938	,,
Sea water, sp. gr. 1·0463	63·5	·903	,,
Sodium Chloride Solution			
Na.Cl. + 10 H_2O	—	·764	Winkelmann
+ 25 H_2O	—	·862	
+ 50 H_2O	—	·918	
+100 H_2O	—	·955	
+200 H_2O	—	·977	
Water	60	·9990	
	100	·9967	Marks and Davis
	200	1·0039	
	300	1·0290	
		Specific heat at constant pressure.	
Air	68	·2415	Swann
	212	·2425	
,,	68 to 824	·2366 (mean)	Holborn and Austin
	68 to 1166	·2429 ,,	
	68 to 1472	·2430 ,,	
Nitrogen	68 to 824	·2419 (mean)	Holborn and Austin
	68 to 1166	·2464 ,,	
	68 to 1472	·2497 ,,	
Oxygen	68 to 824	·2240 (mean)	Holborn and Austin
	68 to 1166	·2300 ,,	
Hydrogen	Room to 392	3·409 (mean)	Regnault
Carbon-monoxide	Room to 378	·245	,,
Carbon-dioxide	32	·1973	Swann
	212	·2213	
,,	68 to 824	·2306 (mean)	Holborn and Austin
	68 to 1166	·2423 ,,	
	68 to 1472	·2486 ,,	
Water vapour (atmos. press.)	230 to t	·446(1+·000053 t) (mean)	Holborn and Henning

TABLE 26

COEFFICIENT OF LINEAR EXPANSION

Substance.	Temperature °F.	Coefficient of Linear * Expansion, per °F.	Authority.
Aluminium	104	·0000128	Fizeau
,,	1112	·0000175	Chatelier
Brass (cast)	32 to 212	·0000104	Smeaton
Copper	104	·00000933	Fizeau
Gold	104	·00000802	,,
Iron (cast)	104	·0000059	,,
,, (soft)	104	·00000675	,,
,, (wrought)	0 to 212	·00000634	Andrews
Lead	104	·0000163	Fizeau
Nickel	104	·0000071	,,
Platinum	104	·000005	,,
Silver	104	·0000107	,,
Steel	104	·00000735	,,
,, (annealed)	104	·0000061	,,
Tin	104	·0000124	,,
Zinc	104	·0000162	,,
Ebonite	77 to 95	·0000468	Kohlrausch
Glass (tube)	32 to 212	·00000462	Smeaton
,, (plate)	32 to 212	·00000495	Lavoisier and Laplace
,, (crown)	32 to 212	·00000499	,, ,,
,, (flint)	122 to 140	·00000432	Pulfrich
,, Jena thermometer 16^{III}	32 to 212	·0000045	Schott
,, ,, ,, 59^{III}	32 to 212	·00000322	,,
Gutta-percha	68	·00011	Russner
Porcelain	68 to 1452	·0000023	Braun
,, (Bayeaux)	1830 to 2550	·00000307	Deville and Troost
Vulcanite	32 to 64	·0000354	Mayer
Mercury	32	·000034	—

* Cubical coefficient of expansion is three times the linear coefficient.

TABLE 27

DENSITY

Substance.	Density at ordinary atmospheric temperature, lbs. per cu. ft.	Substance.	Density at ordinary atmospheric temperature, lbs. per cu. ft.
Aluminium (cast)	160 to 161	Lead (cast)	710
„ (wrought)	165 to 175	„ (wrought)	710
„ (pure)	161	Mercury	848
Brass—		Nickel	540 to 550
(yellow, 70 Cu. + 30 zn.)		Platinum	1320 to 1350
(cast)	527	Silver (cast)	650 to 657
„ „ (rolled)	534	„ (wrought)	661
„ „ (drawn)	542	Tin (white-cast)	455
(red, 90 Cu. + 10 zn.)	536	„ (white-wrought)	455
(white, 50 Cu. + 50 zn.)	511	Zinc (cast)	439 to 447
Copper (cast)	549 to 558	„ (wrought)	449
„ (drawn)	557 to 558	Asbestos	125 to 175
„ (wrought)	552 to 558	Brick	87 to 137
Gold (cast)	1200	Ebonite	72
„ (wrought)	1207	Glass (common)	150 to 175
Iron (pure)	490 to 492	..	..
„ (grey-cast)	439 to 445	Magnesia	200
„ (white-cast)	473 to 482	Paraffin	54 to 57
„ (wrought)	487 to 492	Sea water	64 at 60° F.
„ (steel)	474 to 487	Water	62·37 at 60 °F.

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